Revisiting Cross-Country Correlation Anomalies

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Abstract

It is well known that the two-country real business cycle model (as initially proposed by Backus, Kehoe and Kydland (1992)) has difficulty producing positive cross-country correlations of hours worked and investment. Recently, Baxter and Farr (2005) found that adding endogenous capital utilization rates to a two-country model raises these correlations. However, their results show that for realistic parameter values, the model still cannot match both correlations simultaneously as the cross-country correlation of investment is consistently too low. A first contribution of this paper is to show that learning-by-doing is another mechanism that raises the cross-country correlations of investment and hours. A second contribution is the finding that a two-country model with learning-by-doing and endogenous capital utilization produces positive cross-country correlations of hours and investment that are in the range observed empirically. A third contribution is to show that the model with learning-by-doing and capital utilization can produce positive cross-country correlations of output, consumption, investment and hours when technology shocks are *uncorrelated* across countries and there are international spillovers in the accumulation of organizational capital.

Keywords: International RBC, learning by doing, capital utilization, cross-country correlations

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1. Introduction

The seminal work of Backus, Kehoe and Kydland (1992) on international real business cycles (IRBC) shows that a two-country version of a basic RBC model has great difficulties reproducing observed international co-movements. For example, all models BKK (1995) study generate negative cross-country correlations of investment and hours. While empirically observed correlations differ somewhat depending on the time period, the detrending/filtering method, and countries considered it is fair to say that there is strong evidence that investment and hours are positively correlated across countries. For example, Ambler, Cardia and Zimmermann (2004) look at 190 country pairs and find that on average the cross-country correlations of investment and hours are positive and that at least seventy-five percent of the 190 pairwise correlations are positive. They also find that except for Italy, the pairwise correlations between the United States and other countries are statistically greater than zero.[1]

Little work has been done to extend the model of BKK (1995) to improve its predictions regarding these two correlations. Boileau (2002) finds that a model with trade in capital goods and investment-specific technical change can generate large positive cross-country correlation in investment. A version of his model can also generate a positive cross-country correlation in hours worked. More recently, Baxter and Farr (2005) found that adding endogenous utilization rates to the two-country model raises cross-country correlations of investment and hours. Depending on how costly utilization of capital is (in terms of faster capital depreciation) the two correlations can be made positive. However, their results show that for realistic parameter values, their model still cannot match both correlations simultaneously as the cross-country correlation of investment is consistently too low. Also, Wen (2002) and the results reported in this paper raise doubts about the robustness of Baxter-Farr's results.

Learning-by-doing (LBD)[2] has been shown to help the closed-economy business cycle model to propagate technology shocks (Cooper and Johri, 2002) and monetary shocks (Johri, 2006). In open-economy models, LBD has been shown to help generate persistent movements in the real exchange rate (Johri and Lahiri, 2006) and to help reduce investment volatility and match the trade balance (Luo, 2006).

[1] The other countries considered are Australia, Austria, Canada, France, Germany, Japan, and Swtizerland.

[2] See Arrow (1962) and especially Rosen (1972) for early discussions of learning-by-doing as a by-product of production experience.

We show that adding LBD raises the cross-country correlation of hours and that of investment. LBD slows down the return of hours towards their steady-state level after a productivity shock which promotes a higher cross-country correlation of hours. In a two-country RBC model with international capital mobility, the response of investment is influenced by expected asset returns in addition to consumption smoothing motives. LBD implies that when the home country is hit by a positive technology shock, the marginal product of capital stays high longer than in the baseline model. As a result the home country, taking into account capital adjustment costs, adjusts investment more gradually than in the baseline model. This reduces the movements of capital from the foreign to the home country, helping to make the cross-country correlation of investment less negative.

When we combine LBD and capital utilization in an otherwise standard two-country RBC model, we can generate positive cross-country correlations of investment and hours using parameter values that are commonly used in the macro literature. The interplay of utilization and LBD is actually important for this result. In our model, organizational capital is accumulated as a by-product of production. The higher output, the more organizational capital is created. So when a shock hits a country, the endogenous utilization of capital makes output responds more than in the baseline model resulting in the creation of more organizational capital than in a model where capital utilization is constant. The greater the accumulation of organizational capital, the stronger the effects of LBD which allows the model with both LBD and capital utilization to produce positive cross-country correlations of hours and investment.

The paper is organized as follows. Section 2 presents our two-country model with LBD and endogenous capital utilization rate (the so-called "full" model). Section 3 explains how we solve the model and select parameter values. In section 4 we analyze the dynamic properties of our full model and special cases of it. Section 5 extends the model to include international spillovers in the accumulation of knowledge. Section 6 offers some concluding remarks.

2. Model

Our model builds on the international real business cycle model with incomplete markets proposed by Baxter and Crucini (1995). We extend their model by adding LBD in the spirit of Cooper and Johri (2002) and by adding endogenous capital utilization in the spirit of Greenwood, Hercowitz and Huffman (1988).

There are two countries, "the home country" and "the foreign country". Each country is inhabited by a large number of infinitely lived identical agents. Both countries produce a homogeneous final good which may be used either for consumption or for investment. The final good can be traded freely across the two countries, but trade in financial assets is restricted to a simple non-contingent real bond. We denote all foreign country variables with an asterisk.

The representative agent in the home country seeks to maximize his expected lifetime utility given by:

$$\mathcal{U} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma}}{1-\sigma}, \qquad 1 > \beta > 0,\ 1 > \psi > 0,\ \sigma > 0 \tag{1}$$

where E_t denotes the mathematical expectation operator conditional on the agent's time t information set, C_t denotes consumption, n_t denotes hours worked, β is a discount factor, σ is the coefficient of relative risk aversion, and ψ represents the importance of consumption in the consumption-leisure bundle. As usual, preferences and technologies are the same in both countries.

Output in the home country (Y) is produced using labour (n), a stock of physical capital (K), and a stock of organizational capital (H)

$$Y_t = A_t H_t^{\varepsilon}(u_t K_t)^{\theta} n_t^{\alpha}, \qquad 0 < \varepsilon, \theta, \alpha < 1, \tag{2}$$

where A_t is the (stochastic) exogenous component of total factor productivity and u_t denotes the capital utilization rate.

Similarly, foreign country output is given by the following:

$$Y_t^* = A_t^* H_t^{*\varepsilon}(u_t^* K_t^*)^{\theta} n_t^{*\alpha}, \qquad 0 < \varepsilon, \theta, \alpha < 1 \tag{3}$$

The exogenous process for the productivity shocks is a bivariate autoregressive process (in logs):

$$\begin{bmatrix} \ln A_{t+1} \\ \ln A_{t+1}^* \end{bmatrix} = \begin{bmatrix} \rho & \nu^* \\ \nu & \rho^* \end{bmatrix} \begin{bmatrix} \ln A_t \\ \ln A_t^* \end{bmatrix} + \begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1}^* \end{bmatrix} \tag{4}$$

where ρ and ρ^* measure the persistence of domestic and foreign productivity shocks, ν and ν^* measure the degree of spillovers across countries, and σ_ϵ^2 denotes the variance of innovation ϵ.

The accumulation equation for the stock of physical capital in the home country is

$$K_{t+1} = (1 - \delta_1 u_t^{\delta_2})K_t + I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \bar{\delta}\right)^2 K_t, \qquad 0 \le \phi,\ 0 < \delta_1,\ 1 < \delta_2 \tag{5}$$

where I_t denotes investment made in the home country, $\bar{\delta} \equiv \delta_1 u_{ss}^{\delta_2}$ is the steady-state depreciation rate of physical capital.

For the foreign country, we have:

$$K_{t+1}^* = (1 - \delta_1 u_t^{*\delta_2}) K_t^* + I_t^* - \frac{\phi}{2} \left(\frac{I_t^*}{K_t^*} - \bar{\delta} \right)^2 K_t^*. \tag{6}$$

The accumulation equations for the stock of physical capital (5) and (6) include capital adjustment costs, which are governed by the parameter ϕ, to smooth investment movements. The adjustment costs function is set in such a way that the model with adjustment costs has the same steady state as the model without them.

The organizational capital is accumulated indirectly through the process of production as follows

$$H_{t+1} = H_t^{\gamma} Y_t^{\eta}, \qquad 0 < \gamma < 1, \; 0 < \eta \leq 1 \tag{7}$$

and

$$H_{t+1}^* = H_t^{*\gamma} Y_t^{*\eta}, \qquad 0 < \gamma < 1, \; 0 < \eta \leq 1 \tag{8}$$

where γ reflects the influence of current organizational capital on the accumulation of additional capital and η reflects the influence of current output on the accumulation of organizational capital.

There are at least two ways to think about what constitutes organizational capital. Some, like Rosen (1972), think of it as a firm specific capital good while others focus on specific knowledge embodied in the matches between workers and tasks within the firm. While these differences are important, especially when trying to measure the payments associated with various inputs, they are not crucial to the issues at hand. As a result we do not distinguish between the two.

This specification of how LBD leads to productivity increases draws on early work by Arrow (1962) and Rosen (1972) as well as a large empirical literature dating back roughly a hundred years which documents the pervasive presence of learning effects in virtually every area of the economy. Recent studies include Bahk and Gort (1993), Irwin and Klenow (1994), Jarmin (1994), Benkard (2000), Thompson (2001), Thornton and Thompson (2001) and Cooper and Johri (2002). Our specification is taken from Cooper and Johri (2002) which not only offers a detailed justification for the modeling assumptions but also a number of estimates of the learning technology at different levels of aggregation for the US economy.

The crucial difference between the traditional specification of LBD and this one is that we allow for curvature in the accumulation of knowledge. The traditional specification assumes that organizational capital equals the cumulative sum of all output ever produced by the firm. This modification of the traditional specification of learning has a number of advantages. First, it allows for the sensible idea that production knowledge may become less and less relevant over time as new techniques of production, new product lines and new markets emerge. Second, it allows in a general way for the idea that some match specific knowledge may be lost to the firm as workers leave or get reassigned to new tasks or teams within the firm. In addition, the knowledge accumulated through production experience will be a function of the current vintage of physical capital. The decision to replace physical capital will imply that the existing stock of organizational capital will be less relevant. Third, it allows for the existence of a steady state in which the stock of organizational capital is constant. In contrast, the traditional specification in the empirical LBD literature allows the stock of organizational capital to grow unboundedly. An alternative way to bound learning is to assume that productivity increases due to learning occur for a fixed number of periods. While this may be appropriate for any one task or worker within the firm, we think of the internal context of firms as an environment with an ever changing set of tasks, workers, teams, machines and information. In this context it may be better to model organizational capital as continually accumulating and depreciating.

The restriction $\gamma < 1$ is consistent with the empirical evidence supporting the hypothesis of depreciation of organizational capital often referred to as organizational forgetting. Argote et al. (1990) provide empirical evidence for this hypothesis of organizational forgetting associated with the construction of Liberty Ships during World War II. Similarly, Darr et al. (1995) provide evidence for this hypothesis for pizza franchises and Benkard (2000) provides evidence for organizational forgetting associated with the production of commercial aircraft. One difference between these studies and this paper is that the accumulation technology is log-linear rather than linear. Clarke (2006) shows that the additional curvature in this log-linear technology is unlikely to produce predictions for aggregate variables, in response to a technology shock, considerably different to those associated with a linear technology. It is the implied dynamic structure associated with the accumulation of organizational capital, rather than any functional form assumptions that drives the results in Cooper and Johri (2002).

The financial markets in our world economy are incomplete. More specifically, only one-period risk-free real bonds can be traded. These bonds are traded at price $P_t = (1 + r_{t+1})^{-1}$, where r_{t+1} is the domestic real return on bonds purchased in period t. That is, r_{t+1} is the interest rate linking periods t and $t + 1$. We denote the quantity of discount bonds purchased by residents of the home country in period t by B_{t+1} (each paying one unit of consumption in period $t+1$), the following budget constraint for the representative agent in the home country must be satisfied:

$$C_t + I_t + P_t B_{t+1} = Y_t + B_t. \tag{9}$$

The budget constraint for the foreign country agent is:

$$C_t^* + I_t^* + P_t^* B_{t+1}^* = Y_t^* + B_t^*. \tag{10}$$

We assume that bonds are in zero net supply, so bond market clearing requires

$$B_t + B_t^* = 0. \tag{11}$$

Following Devereux and Smith (2007) we assume there are some frictions in the bond market that create an interest rate differential across countries. Formally we assume

$$1 + r_{t+1} = (1 + r_{t+1}^*)\ e^{-\chi[B_{t+1} - \bar{b}]}, \qquad \chi > 0 \tag{12}$$

where r_t^* is the foreign real interest rate. The "premium" $e^{-\chi[b_{t+1}-\bar{b}]}$ appearing in (12) implies that the further in debt the domestic country gets (the more negative B becomes) the higher the interest rate at home compared to the foreign country. Taking logs, using the approximation that $\log(1+x) \approx x$ for x small, and ignoring a constant term imply

$$r_{t+1} = r_{t+1}^* - \chi B_{t+1}. \tag{13}$$

The bond price version equation (12) is

$$P_t^* = P_t\ e^{-\chi[b_{t+1} - \bar{b}]}.$$

Technically, we need a mechanism to deal with the existence of a unit root in bond accumulation in the incomplete markets economy and our "risk premium" is one way to make the model stationary (see Boileau and Normandin (2006) for more on this issue).

3. Model Solution and Parameter Values

The models we work with in this paper do not have closed-form solutions so we use numerical methods to solve them. The log-linearization method of King, Plosser and Rebelo (2002) is used. To use this method, we assign values to the models' parameters. The values used in our quantitative work are commonly found in the business cycle and international macro literature. Not all parameters are held constant across models so here we detail the strategy we follow. Table 1 displays the parameter values for all models. The "baseline" model refers to a special case of the full model described in section 2 where there is no LBD and no endogenous capital utilization. The "capital utilization" model (the "KU model") refers to a special case of the model described in section 2 where there is no LBD. The "LBD" model refers to a special case of the model described in section 2 where there is no endogenous capital utilization. The system of equations corresponding to all models (and details about how we solved these models) are given in the technical appendix.

The reference period is a quarter.

In all models we set $\beta = 0.99$, $\sigma = 2$. We let ψ adjust so that the fraction of time spent working is always equal to 0.3.

The United States net foreign asset position has dramatically changed since the early 1980s. From the 1950s to the early 1980s, the position (as a percentage of output) was around 10% (see for example Masson (1994)). It has since plunged to around negative 20% (see for example Gourinchas and Rey (2007)). The average over the period 1975-2005 is close to zero (-0.03). Accordingly, we set $\bar{b} = 0$ which implies zero net foreign asset holding in steady state.

An estimate of χ can be found in Lane and Milesi-Ferretti (2001): $\chi = 0.001$.

When capital utilization is included in the model δ_2 is set so that the steady-state capital-output ratio is around ten and the steady-state depreciation rate is 2.5%. Given a value for steady state depreciation, the other parameter appearing in the depreciation function, δ_1, is set to match the empirical observation that the average utilization rate in manufacturing in the US (1972-2007) is 80%.[3] In models where there is no endogenous capital utilization, we set the depreciation rate $\delta = 0.025$.

The capital adjustment cost parameter ϕ is adjusted to insure that the ratio standard deviation of investment over standard deviation of output is 2.66 as in our regression detrended data.

In the models without LBD we set $\theta = 0.36$ and $\alpha = 0.64$. In the model with LBD we do the following. Cooper and Johri's (2002) empirical work show an estimate of 0.26 for ε. If we were to adopt this value for ε and retain the above mentioned values for θ and α in models with LBD, our

[3]Based on the series Capacity Utilization: Manufacturing (series ID CUMFN) issued by the Board of Governors of the Federal Reserve System.

production function would have increasing returns to scale. In order to put models with LBD on the same footing as the other models in terms of returns to scale, we set $\theta = 0.36/1.26$, $\alpha = 0.64/1.26$ and $\varepsilon = 0.26/1.26$ in models with LBD.[4] The value $\varepsilon = 0.26/1.26 = 0.2063$ implies a fifteen percent learning rate which is conservative.

Finally we discuss the parameters related to the technology shocks. The variance of the innovations σ_ϵ^2 adjusts in such a way that each model matches the standard deviation of output in the data (2.52 percent). International spillovers are not estimated precisely (see for example BKK (1992)) so we follow Baxter+Crucini (1995), Baxter+Farr (2005) and others and always set $\nu = \nu^* = 0$.

For the cross-country correlation of shocks we use a recent estimate of $corr(\epsilon, \epsilon^*) = 0.323$ calculated by Boileau and Normandin (2006). Their estimate is slightly larger than the estimate of 0.258 initially found by Backus et al (1992). This is understandable since BKK allowed the shocks process to have non-zero international spillovers when estimating $corr(\epsilon, \epsilon^*)$. Given that our process has zero spillovers we think that Boileau and Normandin's estimate, which was computed imposing zero spillovers, is a more appropriate number.

Setting the degree of persistence in shocks (ρ) is tricky since the Solow residuals, which are commonly used to calibrate ρ, correspond to different quantities across models. In the full model the log of the Solow residuals equals $\ln A + \theta \ln u + \varepsilon \ln H$. In the LBD model the log of the Solow residuals equals $\ln A + \varepsilon \ln H$. In the model with capital utilization only the log of the Solow residuals equals $\ln A + \theta \ln u$. In the baseline model the log of the Solow residuals equals $\ln A$. Several values of ρ appear in the RBC literature. For example, Kydland and Prescott (1982) uses 0.95, King and Rebelo (1999) use 0.98 in their baseline model, BKK (1992, 1995) use 0.906, Baxter and Crucini (1995) use 0.906 and unity, Baxter and Farr (2005) use 0.999, Boileau and Normandin (2006) use 0.924 and 0.995, and Kollmann (1996) use 0.95. Accordingly, we set $\rho = 0.96$ in the baseline model. This implies that the first-order autocorrelation of TFP implied by the baseline model is 0.96. In order to insure that all models are on the same footing in terms of persistence of Solow residuals/TFP we do the following. For each model, we select a value of ρ such that the model delivers an autocorrelation of 0.96 for TFP.

In summary, parameters ϕ, ρ, and σ_ϵ^2 vary across models to insure that all models have the same volatility of output, the same relative volatility of investment and the same persistence in TFP.

[4] Also, without this adjustment in the production function parameters the capital-output ratio would be very large (around fourteen) and much larger than in the other models (ten).

4. Results

4.1 Moments

Table 2 reports moments implied by the full model and special cases of it. Looking at the third and fourth columns of numbers shows that, individually, capital utilization and LBD fail to produce positive cross-country correlation for investment and hours. However, the full model which includes both of these mechanisms produces small positive cross-country correlations for hours and investment (fifth column of numbers). The improvement is especially striking for the investment correlation which goes from -0.40 in the baseline model to 0.30 in the full model. In order to gain some intuition about what is driving these results, we next turn our attention to impulse response functions.

4.2 Hours Dynamics

The figures show impulse response functions (IRFs) produced by our models.[5] To calculate the IRFs we simulate the model while feeding it the following matrix of innovations

$$\begin{bmatrix} \epsilon_t \\ \epsilon_t^* \end{bmatrix}_{t=1}^{\infty} = \begin{bmatrix} 0.01 & 0 & 0 & 0 & \dots \\ 0.01\ corr(\epsilon,\ \epsilon^*) & 0 & 0 & 0 & \dots \end{bmatrix}.$$

That is, the home country experiences a one percent positive shock and the foreign country simultaneously experiences a shock of size $0.01 \times 0.323 = 0.00323$. The four panels in figure 1 show the responses of hours in both countries for all models. Before we explain the responses of hours we make a few remarks about figure 1.

In the baseline model (figure 1A), hours increase in both countries in the period of the shock (period 2 in the graphs). While n stays at its larger value for a few periods before slowly reverting to steady state, n^* peaks in the period of the shock and rapidly falls and undershoots its steady-state level. As a result, n and n^* are negatively correlated in the baseline model (-0.38).

Figure 1B demonstrates the magnifying effect capital utilization has on hours. Both n and n^* increase more than in the baseline model following the shock. As a result n and n^* are further above their steady state level and take longer to converge to it which contributes to raising the correlation of n and n^* (-0.09) in the model with capital utilization.

[5] Unless otherwise indicated, the figures show variables in percent deviation from steady state.

The propagation properties of LBD produce a hump-shaped response of hours in the domestic country (figure 1C) and a response of n^* that converges to steady state at a slower rate than in the baseline model. The stronger response of n and the fact that n^* does not drop below steady state as fast as in the baseline model makes the cross-country correlation of hours less negative in the LBD model (-0.28) than in the baseline model.

Figure 1D shows that the magnification and propagation properties of capital utilization and LBD combine to produce responses of n and n^* that are larger and more persistent (hump-shaped) than in the baseline model. As a result n and n^* are far above steady state for several periods after the shock which produces a positive cross-country correlation of hours (0.04) in the full model.

At this point it is natural to ask what are the driving forces at work behind the responses of n and n^*. We start with the baseline model where the optimality condition for hours (once Lagrange multipliers are eliminated) is

$$\frac{1-\psi}{1-n_t} = \frac{\psi}{C_t} MPn_t \tag{14}$$

where MPn denotes the marginal product of labour. An equivalent condition holds for the foreign country. The left-hand side of the equation represents the cost of working more (less leisure) while the right-hand side represents the benefits of working more (more output so more to consume). Equation (14) imposes restrictions on the joint behaviour of hours, consumption and the marginal product of labour. The linearized version of (14) is

$$\hat{n}_t = (\widehat{mpn}_t - \hat{c}_t)(1-\bar{n})/\bar{n} \tag{14'}$$

where hatted variables are measured in percent deviation from their steady-state values and $\bar{n}$ denotes steady state hours. Clearly, the presence of $\widehat{mpn}_t$ in equation (14′) reflects the existence of a marginal product of labour/wage effect on hours while substitution and wealth effects are captured by changes in $\hat{c}$.

The shocks hitting the two countries (the size of the shock in the foreign country being 32.3% of the size of the shock in the home country) raise wealth in both countries which raises consumption and reduces hours. As (14′) shows, a rise in consumption, everything else equals, reduces hours. Figure 2 shows the responses of C and C^* for the baseline and full models. Clearly in both models and in both countries, the wealth and interest rate effects of the shock on hours which are captured by consumption in (14′) tend to make the responses of n and n^* negative. Given that all models produce positive response of hours in both countries for several periods after the shock, there has to be a dominating effect at works. This effect is the marginal product of labour/wage effect. Therefore, we now focus our attention on the differences in the responses of the marginal product of labour in the baseline and full model to explain why the hours responses in these two models differ so much.

The optimality condition for hours in the full model is

$$\frac{1-\psi}{\psi}\frac{C_t}{1-n_t} = MP^{Y}_{n,t} + \frac{\lambda_{3t}}{\lambda_{1t}} MP^{H'}_{n,t} \tag{15}$$

where MP^Y_N denotes the marginal product of labour in the production of output (it corresponds to MP_N in equation (14)), $MP^{H'}_N$ denotes the marginal product of labour in the production of next period organizational capital, λ_1 is a Lagrange multiplier equal to the marginal utility of consumption, and λ_3 is the Lagrange multiplier associated with the accumulation equation for organizational capital. Equation (15) is derived by combining equations (110) and (112) appearing in the appendix.

Note that given our assumptions about the functional form of the production function and accumulation equations we have

$$MPn^Y_t = \alpha \frac{A_t u^\theta_t H^\varepsilon_t K^\theta_t}{n^{1-\alpha}_t} = \alpha \frac{Y_t}{n_t}, \qquad MP^{H'}_{n,t} = \alpha\eta \frac{{A_t}^{\eta} u_t^{\theta\eta} {K_t}^{\theta\eta} {H_t}^{\eta\varepsilon+\gamma}}{{n_t}^{1-\alpha\eta}} = \alpha\eta \frac{H_{t+1}}{n_t}. \tag{16}$$

Evidently, the labour FOC (14) in the baseline model is a special case of equation (15) where the last term on the right is omitted. Actually, if we define the entire RHS of (15) as TMP_n (the total marginal product of labour) then the linearized version of (15) is

$$\hat{n}_t = (\hat{tmpn}_t - \hat{c}_t)(1-\bar{n})/\bar{n} \tag{15'}$$

which closely resemble (14′). As argued above, the added dynamics that succeed in raising the cross-country correlation of hours in the full model must come from differences in the responses of the marginal product of labour across models. The similarity between the responses of TMP_n and TMP^*_n in figure 3D and those of n and n^* in Figure 1D confirms this inference. Therefore, it is the combined effects of LBD and utilization on the marginal product of labour that allows the full model to produce a positive cross-country correlation of hours. Hence, we need to look at how LBD and endogenous capital utilization change the response of the marginal product of labour.

Let us first look at the first term on the right-hand side of (15), $MP^Y_{n,t}$. Contrary to the baseline model, in the full model the marginal product of labour in the production of output depends on the stock of organizational capital and on the utilization rate of capital (see equation (16)). These additional components of MP^Y_n change its dynamics as shown in figure 3A. Utilization and LBD make the response of MP^Y_n in both countries stronger and more strongly hump shaped than in the baseline model. Just like in the baseline model, MP_n^Y increases in the period of the shock because of the exogenous shocks. However, in the full model, the utilization rate also increases in the period of the shock which makes the initial response larger. Then, in the period after the shock, accumulation of physical and organizational capital kick in which further raises MP_n^Y in both countries. Furthermore, the strong propagation properties of LBD make the response of utilization in the home country slightly hump shaped (which is not the case in the KU model, see figure 3B) which further contributes to the hump shape observed in MP_n^Y. The link between n and MPn^Y imposed by equation (15) implies that the hump-shaped responses in MPn^Y in both countries tend to make the responses of n and n^* hump shaped as well.

The last term in equation (15) comes from the fact that by increasing hours work a country produces more which raises its stock of organizational capital in the following period which then raises future output. From the formula for $MPn^{H'}$ in equation (16) we can make an argument similar to the one above to explain why the responses $MPn^{H'}$ and $MPn^{H'*}$ are hump shaped.

The response of the last term of equation (15) (including the ratio of Lagrange multipliers) is presented in figure 3C. Note that the hump-shaped response of $MPn^{H'}$ ($MPn^{H'*}$) is counterbalanced by the AR(1) response of λ_3/λ_1 (λ_3^*/λ_1^*) so that we do not see hump-shaped responses in figure 3C.

Our analysis implies that the effect of LBD and capital utilization on the responses of hours manifests itself through different marginal product of labour/wage effects in the baseline and full models. The larger and more persistent response of the (total) marginal product of labour in the full model (see figure 3A and 3D) in both countries produce responses of hours in both countries that are further above steady state and persistently so. As a result, the cross-country correlation of hours is positive in the full model. The internal propagation properties of LBD help reduce the speed at which n and n^* move back to steady state after a shock (compare figures 1A and 1C). However, LBD on its own cannot sufficiently smooth out the responses of n and n^* to make the correlation of these two variables positive. Adding capital utilization reinforces the effects of LBD by making the countries accumulate more organizational capital which helps keep hours in both countries above their steady state for many periods after the shock.

4.3 Investment Dynamics

The four panels in Figure 4 show the response of investment in both countries for all models. The baseline model (figure 4A), capital utilization model (figure 4B), and LBD model (figure 4C) all share a common feature: investment moves in opposite directions across countries in the period of the shock which is a determinant factor in the negative correlation of I and I^* produced by these models. As figure 4D clearly shows, the response of investment in the full model is quite different from the other models. This is especially true of the response of I^* which does not fall below steady state at all in the period of the shock and rises well above its steady-state value for several periods after the shock. Clearly, eliminating the initial dip in I^* in the period of the shock greatly contributes to make the cross-country correlation of investment positive in the full model. Accordingly, we focus most of our attention on explaining the different response of I^* in the baseline and full models.

In general, the Euler equation associated with physical capital accumulation when abstracting from capital adjustment costs can be written as follows

$$\lambda_{1t} = \beta E_t \lambda_{1t+1}[1 + R^k_{t+1}] \tag{17}$$

where R^k_{t+1} denotes the total return on investment made in period t in the home country. An equivalent condition holds for the foreign country. Recall that λ_1 is equal to the marginal utility of consumption. In the baseline and full models the definition of R^k is respectively

$$R^k_{t+1} = \theta \frac{Y_{t+1}}{K_{t+1}} - \delta \equiv MPK_{t+1} - \delta \tag{18}$$

$$R^k_{t+1} = \theta \frac{Y_{t+1}}{K_{t+1}} - \delta_1 u^{\delta_2}_{t+1} + \frac{\lambda_{3t+1}}{\lambda_{1t+1}} \theta\eta \frac{H_{t+2}}{K_{t+1}} \equiv MPK^Y_{t+1} - \delta_1 u^{\delta_2}_{t+1} + \frac{\lambda_{3t+1}}{\lambda_{1t+1}} MPK^{H'}_{t+1} \tag{19}$$

The first part of (19) reflects the fact that investing in period t raises output next period by the marginal product of capital in the production of output (MPK^Y). The second part reflects the fact that the depreciation rate of capital depends on the utilization rate, and the third part reflects the fact that investing in period t raises physical capital in period $t+1$ which itself raises organizational capital in period $t+2$ which provides more output in that period ($MPK^{H'}$).

The returns defined in equations (18) and (19) differ in three ways. First, the third term in (19) does not appear in (18) since it comes entirely from the presence of LBD in the full model. Second, the middle term in (19) involves a depreciation function which depends on the utilization rate while the capital depreciation rate is time invariant in (18). Note that the first term in both expressions involves Y which depends on u and H in the full model. So the third difference is that $\theta Y/K$ does not depend on u and H in (18).

In IRBC models like those studied in this paper, flows of goods across countries are largely determined by consumption smoothing motives and by expected asset returns. To explain why investment in the foreign country does not fall in the full model while it does in the baseline model, let's first think about what happens in the foreign country in the period of the shock in our baseline model. The foreign country gets a positive transitory technology shock (recall that this shock is 32.3% as big as the shock simultaneously experienced in the home country). This small shock raises the foreign country's wealth and marginal product of labour/wage. Consumption smoothing implies that the wealth and MP_n effects tend to increase I^*. The small shock also raises the return to capital temporarily in the foreign country (see equation (18)) which also tends to increase I^* (rate of return effect).

Since the foreign country is not closed there is an open-economy consideration to factor in. The relatively larger shock in the home country has effects in the foreign country through the interest rate. As it is the case for the foreign country, the positive shock in the home country raises that country's wealth, marginal product of labour/wage and future expected return to capital which all work to increase investment. Therefore, the home country has strong incentives to invest in capital which leads it to borrow on world markets to finance investment. This puts upward pressures on the interest rate (see the realized interest rate in figure 5A). The foreign country reacts to the significant increase in the interest rate by raising its savings to free up resources to lend to the home country (*i.e.* the foreign country accumulates foreign assets). Part of this adjustment takes the form of lower investment (interest rate effect). From figure 4A, we infer that the interest rate effect on I^* dominate the wage, wealth and rate of return effects since I^* falls on impact.

In the full model, the positive marginal product of labour effect of the shock on I^* is stronger than in the baseline model. We see this by noticing that the response of TMP_n^* is stronger than that of MP_n^* in the period of the shock and keeps on increasing for a few more periods (see figures 3A and 3D). So this is a first factor explaining why I^* is larger in the full model than in the baseline model. While it is difficult to quantify the difference in the wealth effect across the baseline and full models, it is safe to assume that the wealth effect on investment in the full model is no smaller than that in the baseline model since a shock in a given country gives rise to the accumulation of organizational capital which contributes to that country's wealth. Therefore, the combined wealth and marginal product of labour effects work to make the response of I^* more positive in the full model than in the baseline model. As we now argue, the (positive) rate of return effect and (negative) interest effect on I^* are smaller in the full model than in the baseline model.

As explained above, the (negative) interest rate effect on I^* in the baseline model is very strong (stronger than the positive wage, wealth and rate of return effects combined in that model) leading to a fall in I^* in the period of the shock. To explain why this important negative effect is smaller in the full model, we have to look at the home country's desires to borrow from the foreign country since the intensity of this desire drives the response of the interest rate. For the same reasons invoked in the previous paragraph we have that the combined wealth and marginal product of labour effects work to make the response of I more positive in the full model than in the baseline model. Everything else equal, this would mean that the home country would want to borrow even more in the full model than in the baseline model. However, this is not the case. Figure 5B shows that the home country's net foreign assets are significantly less negative in the full model than in the baseline model in the few periods after the shock. As a result, there is less upward pressure on the interest rate in the full model. Figure 5A confirms that the realized interest rate is actually smaller in the full model than in the baseline model. But why is the home country's desire to borrow from abroad smaller in the full model given that the positive wealth and wage effects on I are stronger? We argue that it is because the rate of return effect is smaller in the full model. To see this, first note that the second term in the definition of the return to capital in the full model (19) drags down the return because the capital utilization rate is procyclical. All else equal, this would tend to make the rate of return effect on I smaller. But that cannot be the main driving factor here since the same mechanism is at work in the KU model which predicts a much larger response in I than in the baseline model. Second, figure 5C shows that the marginal product of capital in the production of output (MPK^Y) in the full model stays at its higher level for a few periods after the shock whereas it immediately starts falling in the baseline model. Therefore the home country, expecting that the marginal product of capital will stay high for some times, prefers to increase investment in a more gradual fashion than in the other models (see figure 4) to economize on adjustment costs. This much smaller rate of return effect on I in the full model implies that the home country's incentive to borrow from abroad is significantly smaller in the full model.

In summary, we find that the positive wage and wealth effects on I^* are stronger in the full model and that the negative interest rate effect on I^* is smaller in the full model. These three factors all work together to bring I^* up in the period of the shock. The smaller positive rate of return effect on I^* works against raising I^* up in the period of the shock. Given that we observe a positive response of I^* in figure 4D, we conclude that the smaller positive rate of return effect in the full model is dominated by the other three effects discussed.

The fact that it is necessary to use larger capital adjustment cost in the full model than in the baseline model to match the relative volatility of investment also contributes to slow down the movement in I^* in the period of the shock which helps the full model produce a less negative response of I^*. However, the larger adjustment costs used in the full model are not necessary to explain the positive cross-country correlation of investment in the full model. If we use $\phi = 0.385$ (the value of the capital adjustment cost parameter used in the baseline model) in the full model the cross-country correlation of investment is still positive (0.17).[6] Therefore, it is the combination of LBD and capital utilization that makes the full model able to produce a positive cross-country correlation of investment while capital adjustment costs are instrumental in raising this correlation further.

As figure 4C shows, the combination LBD-capital adjustment costs comes close but do not succeed in eliminating the negative initial response of I^*. The presence of capital utilization in the full model implies that the initial response of output to the shock is larger than in the LBD model which implies that TFP increases more in the full model than in the LBD model in the few periods following the shock since the amount of organizational capital accumulated depends on the level of output. This makes the marginal product of capital high for a longer period in the full model than in the baseline model. As a result, the home country saves on adjustment costs by having investment smoother than in the full model which implies less of a need to borrow so less crowding out of investment in the foreign country.

5. A Model with International Spillovers in Learning

Empirical evidence suggest that TFP is generally positively correlated across industrialized countries. See for example BKK (1995) and Ambler Cardia and Zimmermann (2004) who find fairly large correlations in TFP between the US and Europe (around 0.5). In IRBC models, the positive cross-country correlation of TFP usually comes from having positive international spillovers ($\nu > 0$ in equation (4)) and/or having $corr(\epsilon, \epsilon^*) > 0$. In our model, TFP depends not only on the exogenous shock A but also on the capital utilization rate u and on the stock of organizational capital H. Our production function (2) implies that $\ln TFP = \ln A + \varepsilon \ln H + \theta \ln u$. Therefore, in principle, our full model can produce a positive cross-country correlation of TFP even if the exogenous variables A and A^* are uncorrelated ($corr(\epsilon, \epsilon^*) = 0$ and $\nu = 0$). For this to be the case, the model needs to generate some positive cross-country correlations in utilization rates and/or in the stock of organizational capital.

[6] It is 0.05 for hours.

In the model of Baxter-Farr (2005), the positive cross-country correlation in utilization rate creates a wedge between the cross-country correlation of shocks (0.26) and the cross-country correlation in Solow residuals/TFP (around 0.4). It is not clear whether they would still get a positive cross-country correlation of investment if they calibrated their model so that it matches the cross-country correlation of TFP they document or if their shocks where uncorrelated across countries.

As an extension of our full model we also consider a version of the model where there is international spillovers in the accumulation of organizational capital. In this case, both Y and Y^* enter in the production of knowledge. More specifically, we capture the international spillovers via an externality in the accumulation equation of organizational capital

$$H_{t+1} = H_t^{\gamma} Y_t^{\eta} {Y_t^*}^{\tau}, \qquad 0 < \gamma < 1,\ 0 < \eta \leq 1,\ 0 < \tau \leq 1 \tag{7'}$$

and

$$H_{t+1}^* = {H_t^*}^{\gamma} {Y_t^*}^{\eta} Y_t^{\tau}, \qquad 0 < \gamma < 1,\ 0 < \eta \leq 1,\ 0 < \tau \leq 1 \tag{8'}$$

where τ control the extent of the externality. The accumulation of organizational capital now depends on a weighted average of national outputs which raises the cross-country correlations of H and as a consequence, the cross-country correlation of TFP. We can think of this as a parsimonious way to capture the empirical observation that for most countries productivity growth arises in good part from foreign sources (see Keller (2004) for a survey on technology diffusion) perhaps as a result of technology sourcing for which Griffith, Harrison and van Reenen (2006) find evidence at the firm level.

Parameter τ controls the size of the externality. The value $\tau = 0.165$ was selected so that the model with externality (and zero cross-country correlation in shocks) produces the same cross-country correlation of TFP as the full model.

The last column of table 1 shows the parameter values used in the simulation and the last column of table 2 shows. The moments predicted by the model with externality are remarkably similar to those of the full model.

6. Concluding Remarks

We show that the magnification and propagation properties of LBD and capital utilization work together to improve the two-country RBC model's ability to produce positive cross-country correlation of hours and investment. The analysis presented in this version of the paper is still preliminary. Future versions of the paper will have to pay more attention to predictions other than $corr(n, n^*)$ and $corr(I, I^*)$ and further explore the model with spillovers in learning.

7. Technical Appendix

7.1 System of Equations—Baseline Model

$$\lambda_{1t} = \frac{\psi}{C_t}\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma} \tag{20}$$

$$\lambda_{1t}^{*} = \frac{\psi}{C_t^{*}}\left[C_t^{*\psi}(1-n_t^{*})^{1-\psi}\right]^{1-\sigma} \tag{21}$$

$$\frac{1-\psi}{1-n_t}\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma} = \alpha\lambda_{1t}\frac{Y_t}{n_t} \tag{22}$$

$$\frac{1-\psi}{1-n_t^{*}}\left[C_t^{*\psi}(1-n_t^{*})^{1-\psi}\right]^{1-\sigma} = \alpha\lambda_{1t}^{*}\frac{Y_t^{*}}{n_t^{*}} \tag{23}$$

$$\lambda_{1t} = \lambda_{2t}\left[1-\phi\left(\frac{I_t}{K_t}-\delta\right)\right] \tag{24}$$

$$\lambda_{1t}^{*} = \lambda_{2t}^{*}\left[1-\phi\left(\frac{I_t^{*}}{K_t^{*}}-\delta\right)\right] \tag{25}$$

$$\lambda_{2t} = \beta E_t\left\{\theta\lambda_{1t+1}\frac{Y_{t+1}}{K_{t+1}} + \lambda_{2t+1}\left[1-\delta+\phi\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)^2\right]\right\} \tag{26}$$

$$\lambda_{2t}^{*} = \beta E_t\left\{\theta\lambda_{1t+1}^{*}\frac{Y_{t+1}^{*}}{K_{t+1}^{*}} + \lambda_{2t+1}^{*}\left[1-\delta+\phi\left(\frac{I_{t+1}^{*}}{K_{t+1}^{*}}-\delta\right)\frac{I_{t+1}^{*}}{K_{t+1}^{*}} - \frac{\phi}{2}\left(\frac{I_{t+1}^{*}}{K_{t+1}^{*}}-\delta\right)^2\right]\right\} \tag{27}$$

$$P_t\lambda_{1t} = \beta E_t\lambda_{1t+1} \tag{28}$$

$$P_t^{*}\lambda_{1t}^{*} = \beta E_t\lambda_{1t+1}^{*} \tag{29}$$

$$C_t + I_t + P_tB_{t+1} = Y_t + B_t \tag{30}$$

$$C_t^{*} + I_t^{*} + P_t^{*}B_{t+1}^{*} = Y_t^{*} + B_t^{*} \tag{31}$$

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t}-\delta\right)^2 K_t \tag{32}$$

$$K^*_{t+1} = (1-\delta)K^*_t + I^*_t - \frac{\phi}{2}\left(\frac{I^*_t}{K^*_t} - \delta\right)^2 K^*_t \tag{33}$$

$$Y_t = A_t K_t^\theta n_t^\alpha \tag{34}$$

$$Y^*_t = A^*_t {K^*_t}^\theta {n^*_t}^\alpha \tag{35}$$

$$B_t + B^*_t = 0 \tag{36}$$

$$P^*_t = P_t \; e^{-\chi[B_{t+1}-\bar{b}]} \tag{37}$$

There are 18 endogenous variables: $C, n, Y, K, I, B, P, \lambda_1, \lambda_2$, for both countries. There are 18 equations: (20) to (37). We work with a model where the steady state is symmetric ($B = B^* = 0$) and where adjustment costs are assumed to be zero.

The model will be solved using the KPR method. Note that adding up the two budget constraints and imposing bonds market clearing implies

$$C_t + C^*_t + I_t + I^*_t + B_{t+1}(P_t - P^*_t) = Y_t + Y^*_t$$

using (37) we get

$$C_t + C^*_t + I_t + I^*_t + B_{t+1}P_t(1 - e^{-\chi[B_{t+1}-\bar{b}]}) = Y_t + Y^*_t.$$

As long as we have a steady state bond holding of zero, the linearized version of the above equation is identical to the linearized version of the usual resource constraint

$$C_t + C^*_t + I_t + I^*_t = Y_t + Y^*_t. \tag{38}$$

Recognizing that (linearized versions of) the two budget constraints (30) (31) the asset equilibrium condition (36) and the resource constraint (38) are not independent, we follow what Baxter and Crucini (1995) did when solving the model by KPR. That is (1) we drop the foreign budget constraint from the system and include the resource constraint, (2) we make the Lagrange multiplier associated with the home budget constraint a co-state variable. In addition, we simplify the system by eliminating prices P and P^* by using (28) into (30) and by combining (28), (29) and (37). As a result, the system we work with includes 15 equations: (20)-(27), (32)-(35), (38)

$$C_t + I_t + \beta E_t \frac{\lambda_{1t+1}}{\lambda_{1t}} B_{t+1} = Y_t + B_t \tag{39}$$

and

$$E_t \frac{\lambda^*_{1t+1}}{\lambda^*_{1t}} = E_t \frac{\lambda_{1t+1}}{\lambda_{1t}}\ e^{-\chi[B_{t+1}-\bar{b}]} \tag{40}$$

The variables in the system are the state variables $[K_t,\, K^*_t,\, B_t]$, the co-state variables $[\lambda_{2t}, \lambda^*_{2t}, \lambda_{1t}]$ and the control variables $[C_t, C^*_t, N_t, N^*_t, I_t, I^*_t, Y_t, Y^*_t, \lambda^*_{1t}]$

7.2 System of Equations—LBD Model

$$\lambda_{1t} = \frac{\psi}{C_t}\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma} \tag{41}$$

$$\lambda_{1t}^* = \frac{\psi}{C_t^*}\left[C_t^{*\psi}(1-n_t^*)^{1-\psi}\right]^{1-\sigma} \tag{42}$$

$$\frac{1-\psi}{1-n_t}\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma} = \alpha\lambda_{1t}\frac{Y_t}{n_t} + \alpha\eta\lambda_{3t}\frac{H_t^{\gamma}Y_t^{\eta}}{n_t} \tag{43}$$

$$\frac{1-\psi}{1-n_t^*}\left[C_t^{*\psi}(1-n_t^*)^{1-\psi}\right]^{1-\sigma} = \alpha\lambda_{1t}^*\frac{Y_t^*}{n_t^*} + \alpha\eta\lambda_{3t}^*\frac{H_t^{*\gamma}Y_t^{*\eta}}{n_t^*} \tag{44}$$

$$\lambda_{1t} = \lambda_{2t}\left[1-\phi\left(\frac{I_t}{K_t}-\delta\right)\right] \tag{45}$$

$$\lambda_{1t}^* = \lambda_{2t}^*\left[1-\phi\left(\frac{I_t^*}{K_t^*}-\delta\right)\right] \tag{46}$$

$$\lambda_{2t} = \beta E_t\left\{\theta\lambda_{1t+1}\frac{Y_{t+1}}{K_{t+1}} + \lambda_{2t+1}\left[1-\delta+\phi\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)\frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\delta\right)^2\right]... \right. \tag{47}$$

$$\left. ... + \lambda_{3t+1}\left[\theta\eta\frac{H_{t+1}^{\gamma}Y_{t+1}^{\eta}}{K_{t+1}}\right]\right\}$$

$$\lambda_{2t}^* = \beta E_t\left\{\theta\lambda_{1t+1}^*\frac{Y_{t+1}^*}{K_{t+1}^*} + \lambda_{2t+1}^*\left[1-\delta+\phi\left(\frac{I_{t+1}^*}{K_{t+1}^*}-\delta\right)\frac{I_{t+1}^*}{K_{t+1}^*} - \frac{\phi}{2}\left(\frac{I_{t+1}^*}{K_{t+1}^*}-\delta\right)^2\right]... \right. \tag{48}$$

$$\left. ... + \lambda_{3t+1}^*\left[\theta\eta\frac{{H_{t+1}^*}^{\gamma}{Y_{t+1}^*}^{\eta}}{K_{t+1}^*}\right]\right\}$$

$$\lambda_{3t} = \beta E_t\left\{\varepsilon\lambda_{1t+1}\frac{Y_{t+1}}{H_{t+1}} + (\gamma+\varepsilon\eta)\lambda_{3t+1}H_{t+1}^{\gamma-1}Y_{t+1}^{\eta}\right\} \tag{49}$$

$$\lambda_{3t}^* = \beta E_t\left\{\varepsilon\lambda_{1t+1}^*\frac{Y_{t+1}^*}{H_{t+1}^*} + (\gamma+\varepsilon\eta)\lambda_{3t+1}^*{H_{t+1}^*}^{\gamma-1}{Y_{t+1}^*}^{\eta}\right\} \tag{50}$$

$$P_t\lambda_{1t} = \beta E_t\lambda_{1t+1} \tag{51}$$

$$P_t^*\lambda_{1t}^* = \beta E_t\lambda_{1t+1}^* \tag{52}$$

$$C_t + I_t + P_tB_{t+1} = Y_t + B_t \tag{53}$$

$$C_t^* + I_t^* + P_t^* B_{t+1}^* = Y_t^* + B_t^* \tag{54}$$

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \tag{55}$$

$$K_{t+1}^* = (1-\delta)K_t^* + I_t^* - \frac{\phi}{2}\left(\frac{I_t^*}{K_t^*} - \delta\right)^2 K_t^* \tag{56}$$

$$H_{t+1} = H_t^\gamma Y_t^\eta \tag{57}$$

$$H_{t+1}^* = {H_t^*}^\gamma {Y_t^*}^\eta \tag{58}$$

$$Y_t = A_t H_t^\varepsilon K_t^\theta n_t^\alpha \tag{59}$$

$$Y_t^* = A_t^* {H_t^*}^\varepsilon {K_t^*}^\theta {n_t^*}^\alpha \tag{60}$$

$$B_t + B_t^* = 0 \tag{61}$$

$$P_t^* = P_t \; e^{-\chi[B_{t+1} - \bar{b}]} \tag{62}$$

There are 22 endogenous variables: $C, n, Y, H, K, I, B, P, \lambda_1, \lambda_2, \lambda_3$ for both countries. There are 22 equations: (41) to (62). As in the baseline model, we assume $\bar{b} = 0$ (no asset holding in steady state) and no adjustment cost in steady state.

From (51) we get

$$P_{ss} = \beta. \tag{63}$$

Combining (45), (47), (49), and (57) we get

$$\left(\frac{Y}{K}\right)_{ss} = \frac{1-\beta(1-\delta)}{\theta\beta\left[1 + \frac{\eta\beta\varepsilon}{1-\beta(\gamma+\varepsilon\eta)}\right]} \tag{64}$$

Combining (57) and (59) we get

$$Y_{ss} = \left[\bar{A}\left(\frac{K}{Y}\right)_{ss}^\theta n_{ss}^\alpha\right]^{\frac{1-\gamma}{(1-\gamma)(1-\theta)-\eta\varepsilon}}. \tag{65}$$

Using previously derived relations/equations we get

$$K_{ss} = Y_{ss} \left(\frac{K}{Y}\right)_{ss} \tag{66}$$

$$I_{ss} = \delta K_{ss} \tag{67}$$

$$H_{ss} = Y_{ss}^{\frac{\eta}{1-\gamma}}. \tag{68}$$

Budget constraint (53) and $\bar{b} = 0$ imply

$$C_{ss} = Y_{ss} - I_{ss} \tag{69}$$

$$\psi^* = \frac{1}{1 + \alpha \frac{Y_{ss}(1-n_{ss})}{C_{ss} n_{ss}} \left[1 + \frac{\eta\beta\varepsilon}{1-\beta(\gamma+\eta\varepsilon)}\right]} \tag{70}$$

$$\lambda_{1ss} = \frac{\psi^*}{C_{ss}} \left[C_{ss}^{\psi^*} (1 - n_{ss})^{1-\psi^*}\right]^{1-\sigma} \tag{71}$$

$$\lambda_{2ss} = \lambda_{1ss} \tag{72}$$

$$\lambda_{3ss} = \frac{\beta\varepsilon}{1 - \beta(\gamma + \varepsilon\eta)} \lambda_{1ss} \frac{Y_{ss}}{H_{ss}} \tag{73}$$

The model is solved by KPR. We follow the strategy used in solving the baseline model: (1) we drop the foreign budget constraint from the system and include the resource constraint, (2) we make the Lagrange multiplier associated with the home budget constraint a co-state variable. In addition, we simplify the system by eliminating prices P and P^* by using (51) into (53) and by combining (51), (52) and (62). As a result, the system we work with includes 19 equations: (41)-(50), (55)-(60), (38)

$$C_t + I_t + \beta E_t \frac{\lambda_{1t+1}}{\lambda_{1t}} B_{t+1} = Y_t + B_t \tag{74}$$

and

$$E_t \frac{\lambda^*_{1t+1}}{\lambda^*_{1t}} = E_t \frac{\lambda_{1t+1}}{\lambda_{1t}}\ e^{-\chi[B_{t+1}-\bar{b}]} \tag{75}$$

The variables in the system are the state variables $[K_t,\ K_t^*,\ H_t,\ H_t^*,\ B_t]$, the co-state variables $[\lambda_{2t}, \lambda^*_{2t}, \lambda_{1t}, \lambda_{3t}, \lambda^*_{3t},]$ and the control variables $[C_t, C_t^*, N_t, N_t^*, I_t, I_t^*, Y_t, Y_t^*, \lambda^*_{1t}]$

7.3 System of Equations—Model with Capital Utilization

$$\lambda_{1t} = \frac{\psi}{C_t}\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma} \tag{76}$$

$$\lambda_{1t}^* = \frac{\psi}{C_t^*}\left[C_t^{*\psi}(1-n_t^*)^{1-\psi}\right]^{1-\sigma} \tag{77}$$

$$\frac{1-\psi}{1-n_t}\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma} = \alpha\lambda_{1t}\frac{Y_t}{n_t} \tag{78}$$

$$\frac{1-\psi}{1-n_t^*}\left[C_t^{*\psi}(1-n_t^*)^{1-\psi}\right]^{1-\sigma} = \alpha\lambda_{1t}^*\frac{Y_t^*}{n_t^*} \tag{79}$$

$$\lambda_{1t} = \lambda_{2t}\left[1-\phi\left(\frac{I_t}{K_t}-\bar{\delta}\right)\right] \tag{80}$$

$$\lambda_{1t}^* = \lambda_{2t}^*\left[1-\phi\left(\frac{I_t^*}{K_t^*}-\bar{\delta}\right)\right] \tag{81}$$

$$\lambda_{2t} = \beta E_t\left\{\theta\lambda_{1t+1}\frac{Y_{t+1}}{K_{t+1}} + \lambda_{2t+1}\left[1-\delta_1 u_{t+1}^{\delta_2} + \phi\left(\frac{I_{t+1}}{K_{t+1}}-\bar{\delta}\right)\frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\bar{\delta}\right)^2\right]\right\} \tag{82}$$

$$\lambda_{2t}^* = \beta E_t\left\{\theta\lambda_{1t+1}^*\frac{Y_{t+1}^*}{K_{t+1}^*} + \lambda_{2t+1}^*\left[1-\delta_1 {u_{t+1}^*}^{\delta_2} + \phi\left(\frac{I_{t+1}^*}{K_{t+1}^*}-\bar{\delta}\right)\frac{I_{t+1}^*}{K_{t+1}^*} - \frac{\phi}{2}\left(\frac{I_{t+1}^*}{K_{t+1}^*}-\bar{\delta}\right)^2\right]\right\} \tag{83}$$

$$\theta\lambda_{1t}\frac{Y_t}{u_t} - \delta_1\delta_2\lambda_{2t}u_t^{\delta_2-1}K_t = 0 \tag{84}$$

$$\theta\lambda_{1t}^*\frac{Y_t^*}{u_t^*} - \delta_1\delta_2\lambda_{2t}^*{u_t^*}^{\delta_2-1}K_t^* = 0 \tag{85}$$

$$P_t\lambda_{1t} = \beta E_t\lambda_{1t+1} \tag{86}$$

$$P_t^*\lambda_{1t}^* = \beta E_t\lambda_{1t+1}^* \tag{87}$$

$$C_t + I_t + P_tB_{t+1} = Y_t + B_t \tag{88}$$

$$C_t^* + I_t^* + P_t^*B_{t+1}^* = Y_t^* + B_t^* \tag{89}$$

$$K_{t+1} = (1 - \delta_1 u_t^{\delta_2}) K_t + I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \bar{\delta} \right)^2 K_t \tag{90}$$

$$K_{t+1}^* = (1 - \delta_1 u_t^{*\delta_2}) K_t^* + I_t^* - \frac{\phi}{2} \left(\frac{I_t^*}{K_t^*} - \bar{\delta} \right)^2 K_t^* \tag{91}$$

$$Y_t = A_t (u_t K_t)^\theta n_t^\alpha \tag{92}$$

$$Y_t^* = A_t^* (u_t^* K_t^*)^\theta n_t^{*\alpha} \tag{93}$$

$$B_t + B_t^* = 0 \tag{94}$$

$$P_t^* = P_t \; e^{-\chi[B_{t+1} - \bar{b}]} \tag{95}$$

There are 20 endogenous variables: $C, n, Y, u, K, I, B, P, \lambda_1, \lambda_2$ for both countries. There are 20 equations: (76) to (95). As is customary, we work with a steady state where there are no adjustment costs. This implies the steady-state depreciation rate (denoted $\bar{\delta}$) is

$$\delta_1 u^{\delta_2} \equiv \bar{\delta} = \frac{I}{K}. \tag{96}$$

The equations below define steady-state expressions for the endogenous variables. Note that my derivation of the steady state assumes we know steady-state hours and solve for the value of ψ that is consistent with the calibrated value of n (denoted $\bar{n}$).

From (122), (123) and (133) we get

$$P_{ss} = P_{ss}^* = \beta, \qquad B_{ss} = \bar{b}. \tag{97}$$

Combining (80), (82), and (84) we get

$$\left(\frac{Y}{K} \right)_{ss} = \frac{\delta_2 (1 - \beta)}{\theta \beta (\delta_2 - 1)} \tag{98}$$

$$\delta_1 u^{\delta_2} = \frac{1 - \beta}{\beta(\delta_2 - 1)} \equiv \bar{\delta} \tag{99}$$

$$u_{ss}=\left[\frac{1-\beta}{\beta\delta_1(\delta_2-1)}\right]^{\frac{1}{\delta_2}} \tag{100}$$

Then (92) implies

$$K_{ss}=\left[\bar{A}u_{ss}^{\theta}\bar{n}^{\alpha}\left(\frac{k}{Y}\right)_{ss}\right]^{\frac{1}{1-\theta}} \tag{101}$$

Evidently

$$Y_{ss}=K_{ss}\left(\frac{Y}{K}\right)_{ss}. \tag{102}$$

From (90) we have

$$I_{ss}=\bar{\delta}K_{ss} \tag{103}$$

Budget constraint (88) and $\bar{b}=0$ imply

$$C_{ss}=Y_{ss}-I_{ss} \tag{104}$$

We now solve for the value of ψ that is consistent with hour calibration of steady-state hours using equations (76) and (78)

$$\psi^{*}=\frac{1}{1+\alpha\frac{Y_{ss}(1-\bar{n})}{C_{ss}\bar{n}}} \tag{105}$$

then (76) implies

$$\lambda_{1ss}=\frac{\psi^{*}}{C_{ss}}\left[C_{ss}^{\psi^{*}}(1-\bar{n})^{1-\psi^{*}}\right]^{1-\sigma} \tag{106}$$

(80) implies

$$\lambda_{2ss}=\lambda_{1ss} \tag{107}$$

The model is solved by KPR. We follow the strategy used in solving the baseline model: (1) we drop the foreign budget constraint from the system and include the resource constraint, (2) we make the Lagrange multiplier associated with the home budget constraint a co-state variable. In addition, we simplify the system by eliminating prices P and P^{*} by using (86) into (88) and by combining (86), (87) and (95). As a result, the system we work with includes 17 equations: (76)-(85), (90)-(93), (38),

$$C_t+I_t+\beta E_t\frac{\lambda_{1t+1}}{\lambda_{1t}}B_{t+1}=Y_t+B_t \tag{108}$$

and

$$E_t\frac{\lambda_{1t+1}^{*}}{\lambda_{1t}^{*}}=E_t\frac{\lambda_{1t+1}}{\lambda_{1t}}\ e^{-\chi[B_{t+1}-\bar{b}]} \tag{109}$$

The variables in the system are the state variables $[K_t,\ K_t^{*},\ B_t]$, the co-state variables $[\lambda_{2t},\lambda_{2t}^{*},\lambda_{1t}]$ and the control variables $[C_t,C_t^{*},N_t,N_t^{*},I_t,I_t^{*},Y_t,Y_t^{*},\lambda_{1t}^{*},u_t,u_t^{*}]$

7.4 System of Equations—Full Model

$$\lambda_{1t} = \frac{\psi}{C_t}\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma} \tag{110}$$

$$\lambda_{1t}^* = \frac{\psi}{C_t^*}\left[C_t^{*\psi}(1-n_t^*)^{1-\psi}\right]^{1-\sigma} \tag{111}$$

$$\frac{1-\psi}{1-n_t}\left[C_t^{\psi}(1-n_t)^{1-\psi}\right]^{1-\sigma} = \alpha\lambda_{1t}\frac{Y_t}{n_t} + \alpha\eta\lambda_{3t}\frac{H_t^{\gamma}Y_t^{\eta}}{n_t} \tag{112}$$

$$\frac{1-\psi}{1-n_t^*}\left[C_t^{*\psi}(1-n_t^*)^{1-\psi}\right]^{1-\sigma} = \alpha\lambda_{1t}^*\frac{Y_t^*}{n_t^*} + \alpha\eta\lambda_{3t}^*\frac{H_t^{*\gamma}Y_t^{*\eta}}{n_t^*} \tag{113}$$

$$\lambda_{1t} = \lambda_{2t}\left[1-\phi\left(\frac{I_t}{K_t}-\bar{\delta}\right)\right] \tag{114}$$

$$\lambda_{1t}^* = \lambda_{2t}^*\left[1-\phi\left(\frac{I_t^*}{K_t^*}-\bar{\delta}\right)\right] \tag{115}$$

$$\lambda_{2t} = \beta E_t\left\{\theta\lambda_{1t+1}\frac{Y_{t+1}}{K_{t+1}} + \lambda_{2t+1}\left[1-\delta_1 u_{t+1}^{\delta_2} + \phi\left(\frac{I_{t+1}}{K_{t+1}}-\bar{\delta}\right)\frac{I_{t+1}}{K_{t+1}} - \frac{\phi}{2}\left(\frac{I_{t+1}}{K_{t+1}}-\bar{\delta}\right)^2\right]... \right. \tag{116}$$

$$\left. ... + \lambda_{3t+1}\left[\theta\eta\frac{H_{t+1}^{\gamma}Y_{t+1}^{\eta}}{K_{t+1}}\right]\right\}$$

$$\lambda_{2t}^* = \beta E_t\left\{\theta\lambda_{1t+1}^*\frac{Y_{t+1}^*}{K_{t+1}^*} + \lambda_{2t+1}^*\left[1-\delta_1 {u_{t+1}^*}^{\delta_2} + \phi\left(\frac{I_{t+1}^*}{K_{t+1}^*}-\bar{\delta}\right)\frac{I_{t+1}^*}{K_{t+1}^*} - \frac{\phi}{2}\left(\frac{I_{t+1}^*}{K_{t+1}^*}-\bar{\delta}\right)^2\right]... \right. \tag{117}$$

$$\left. ... + \lambda_{3t+1}^*\left[\theta\eta\frac{{H_{t+1}^*}^{\gamma}{Y_{t+1}^*}^{\eta}}{K_{t+1}^*}\right]\right\}$$

$$\lambda_{3t} = \beta E_t\left\{\varepsilon\lambda_{1t+1}\frac{Y_{t+1}}{H_{t+1}} + (\gamma+\varepsilon\eta)\lambda_{3t+1}H_{t+1}^{\gamma-1}Y_{t+1}^{\eta}\right\} \tag{118}$$

$$\lambda_{3t}^* = \beta E_t\left\{\varepsilon\lambda_{1t+1}^*\frac{Y_{t+1}^*}{H_{t+1}^*} + (\gamma+\varepsilon\eta)\lambda_{3t+1}^*{H_{t+1}^*}^{\gamma-1}{Y_{t+1}^*}^{\eta}\right\} \tag{119}$$

$$\theta\lambda_{1t}\frac{Y_t}{u_t} - \delta_1\delta_2\lambda_{2t}u_t^{\delta_2-1}K_t + \theta\eta\lambda_{3t}\frac{H_t^{\gamma}Y_t^{\eta}}{u_t} = 0 \tag{120}$$

$$\theta\lambda_{1t}^*\frac{Y_t^*}{u_t^*} - \delta_1\delta_2\lambda_{2t}^*u_t^{*\delta_2-1}K_t^* + \theta\eta\lambda_{3t}^*\frac{H_t^{*\gamma}Y_t^{*\eta}}{u_t^*} = 0 \tag{121}$$

$$P_t \lambda_{1t} = \beta E_t \lambda_{1t+1} \tag{122}$$

$$P_t^* \lambda_{1t}^* = \beta E_t \lambda_{1t+1}^* \tag{123}$$

$$C_t + I_t + P_t B_{t+1} = Y_t + B_t \tag{124}$$

$$C_t^* + I_t^* + P_t^* B_{t+1}^* = Y_t^* + B_t^* \tag{125}$$

$$K_{t+1} = (1 - \delta_1 u_t^{\delta_2}) K_t + I_t - \frac{\phi}{2} \left(\frac{I_t}{K_t} - \bar{\delta} \right)^2 K_t \tag{126}$$

$$K_{t+1}^* = (1 - \delta_1 u_t^{*\delta_2}) K_t^* + I_t^* - \frac{\phi}{2} \left(\frac{I_t^*}{K_t^*} - \bar{\delta} \right)^2 K_t^* \tag{127}$$

$$H_{t+1} = H_t^{\gamma} Y_t^{\eta} \tag{128}$$

$$H_{t+1}^* = H_t^{*\gamma} Y_t^{*\eta} \tag{129}$$

$$Y_t = A_t H_t^{\varepsilon} (u_t K_t)^{\theta} n_t^{\alpha} \tag{130}$$

$$Y_t^* = A_t^* H_t^{*\varepsilon} (u_t^* K_t^*)^{\theta} n_t^{*\alpha} \tag{131}$$

$$B_t + B_t^* = 0 \tag{132}$$

$$P_t^* = P_t \ e^{-\chi[B_{t+1} - \bar{b}]} \tag{133}$$

There are 24 endogenous variables: $C, n, Y, H, u, K, I, B, P, \lambda_1, \lambda_2, \lambda_3$ for both countries. There are 24 equations: (110) to (133). As is customary, we work with a steady state where there are no adjustment costs. This implies the steady-state depreciation rate (denoted $\bar{\delta}$) is

$$\delta_1 u^{\delta_2} \equiv \bar{\delta} = \frac{I}{K}. \tag{134}$$

The equations below define steady-state expressions for the endogenous variables. Note that my derivation of the steady state assumes we know steady-state hours and solve for the value of ψ that is consistent with the calibrated value of n (denoted $\bar{n}$).

From (122), (123) and (133) we get

$$P_{ss} = P^*_{ss} = \beta, \qquad B_{ss} = \bar{b}. \tag{135}$$

Combining (114), (116), (118), and (120) we get

$$\left(\frac{Y}{K}\right)_{ss} = \frac{\delta_2(1-\beta)}{\theta\beta(\delta_2 - 1)\left[1 + \frac{\eta\beta\varepsilon}{1-\beta(\gamma+\varepsilon\eta)}\right]} \tag{136}$$

$$\delta_1 u^{\delta_2} = \frac{\theta}{\delta_2}\frac{Y}{K}\left[1 + \frac{\eta\beta\varepsilon}{1-\beta(\gamma+\varepsilon\eta)}\right] = \frac{1-\beta}{\beta(\delta_2-1)} \equiv \bar{\delta} \tag{137}$$

$$u_{ss} = \left(\frac{\bar{\delta}}{\delta_1}\right)^{\frac{1}{\delta_2}} = \left[\frac{1-\beta}{\beta\delta_1(\delta_2-1)}\right]^{\frac{1}{\delta_2}} \tag{138}$$

Combining (128) and (130) we get

$$Y_{ss} = \left[\bar{A}u^{\theta}_{ss}\left(\frac{K}{Y}\right)^{\theta}_{ss}\bar{n}^{\alpha}\right]^{\frac{1-\gamma}{(1-\gamma)(1-\theta)-\eta\varepsilon}}. \tag{139}$$

Evidently

$$K_{ss} = Y_{ss}\left(\frac{K}{Y}\right)_{ss}. \tag{140}$$

From (126) we have

$$I_{ss} = \bar{\delta}K_{ss} \tag{141}$$

From (128) we have

$$H_{ss} = Y_{ss}^{\frac{\eta}{1-\gamma}}. \tag{142}$$

Budget constraint (124) and $\bar{b} = 0$ imply

$$C_{ss} = Y_{ss} - I_{ss} \tag{143}$$

We now solve for the value of ψ that is consistent with hour calibration of steady-state hours using equations (110), (112) and (118)

$$\psi^* = \frac{1}{1 + \alpha\frac{Y_{ss}(1-\bar{n})}{C_{ss}\bar{n}}\left[1 + \frac{\eta\beta\varepsilon}{1-\beta(\gamma+\eta\varepsilon)}\right]} \tag{144}$$

then (110) implies

$$\lambda_{1ss} = \frac{\psi^*}{C_{ss}}\left[C_{ss}^{\psi^*}(1-\bar{n})^{1-\psi^*}\right]^{1-\sigma} \tag{145}$$

(114) implies

$$\lambda_{2ss} = \lambda_{1ss} \tag{146}$$

and finally, (118) implies

$$\lambda_{3ss} = \frac{\beta\varepsilon}{1-\beta(\gamma+\varepsilon\eta)}\lambda_{1ss}\frac{Y_{ss}}{H_{ss}} \tag{147}$$

The model is solved by KPR. We follow the strategy used in solving the baseline model: (1) we drop the foreign budget constraint from the system and include the resource constraint, (2) we make the Lagrange multiplier associated with the home budget constraint a co-state variable. In addition, we simplify the system by eliminating prices P and P^* by using (122) into (124) and by combining (122), (123) and (133). As a result, the system we work with includes 21 equations: (110)-(121), (126)-(131), (38),

$$C_t + I_t + \beta E_t\frac{\lambda_{1t+1}}{\lambda_{1t}}B_{t+1} = Y_t + B_t \tag{148}$$

and

$$E_t\frac{\lambda^*_{1t+1}}{\lambda^*_{1t}} = E_t\frac{\lambda_{1t+1}}{\lambda_{1t}}\ e^{-\chi[B_{t+1}-\bar{b}]} \tag{149}$$

The variables in the system are the state variables $[K_t,\ K^*_t,\ H_t,\ H^*_t,\ B_t]$, the co-state variables $[\lambda_{2t}, \lambda^*_{2t}, \lambda_{1t}, \lambda_{3t}, \lambda^*_{3t},]$ and the control variables $[C_t, C^*_t, N_t, N^*_t, I_t, I^*_t, Y_t, Y^*_t, \lambda^*_{1t}, u_t, u^*_t]$

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Table 1: Parameter Values

Parameter	**Baseline**	**K util.**	**LBD**	**LBD and K util.**	**LBD and K util.**
β	0.99	0.99	0.99	0.99	0.99
σ	2	2	2	2	2
ψ	0.3324	0.3324	0.3340	0.3339	0.3339
$\bar{b}$	0	0	0	0	0
χ	0.001	0.001	0.001	0.001	0.001
δ	0.025	na	0.025	na	na
δ_1	na	0.034	na	0.034	0.034
δ_2	na	1.404	na	1.404	1.404
ϕ	0.385	0.053	0.71	0.65	0.58
θ	0.36	0.36	0.36/1.26	0.36/1.26	0.36/1.26
α	0.64	0.64	0.64/1.26	0.64/1.26	0.64/1.26
ε	na	na	0.26/1.26	0.26/1.26	0.26/1.26
γ	na	na	0.5	0.5	0.5
η	na	na	0.5	0.5	0.5
τ	na	na	na	na	**0.165**
$\rho = \rho^*$	0.96	0.98	0.92	0.925	0.925
$\nu = \nu^*$	0	0	0	0	0
σ_ϵ^2	0.00393^2	0.00252^2	0.00475^2	0.0037^2	0.00358^2
$corr(\epsilon, \epsilon^*)$	0.323	0.323	0.323	0.323	0
K/Y	10.26	10.26	10.2	10.2	10.2

Table 2 : Moments

Moment	**Data**	**Baseline**	**K Util.**	**LBD**	**LBD and K util.**	**LBD and K util.**
Standard Deviations (SD)						
$SD(Y)^{\dagger}$	2.52	2.52	2.52	2.52	2.52	2.52
$SD(C)/SD(Y)$	0.95	0.70	0.73	0.62	0.51	0.51
$SD(n)/SD(Y)$	0.78	0.34	0.34	0.39	0.45	0.45
$SD(I)/SD(Y)^{\dagger}$	2.66	2.66	2.66	2.66	2.66	2.66
$SD(TB/Y)$	0.31	1.26	1.10	1.12	0.61	0.57
Cross-Country Correlations						
Y	0.63	0.14	0.23	0.17	0.24	0.25
C	0.42	0.35	0.32	0.43	0.41	0.41
n	0.27	-0.38	-0.09	-0.28	0.04	0.05
I	0.10	-0.40	-0.26	-0.19	0.30	0.28
TFP		0.32	0.37	0.28	0.28	0.28
Autocorrelations						
Y	0.93	0.98	0.98	0.97	0.96	0.97
C	0.95	0.99	0.99	0.99	0.98	0.99
n	0.96	0.97	0.97	0.96	0.96	0.96
I	0.94	0.83	0.78	0.86	0.93	0.93
$TFP^{\dagger}$		0.96	0.96	0.96	0.96	0.96
TB/Y	1	0.80	0.65	0.85	0.91	0.92

Notes: † denotes a moment that is targeted in the calibration (see section XXX of the paper).

Figure 1A: Baseline Model

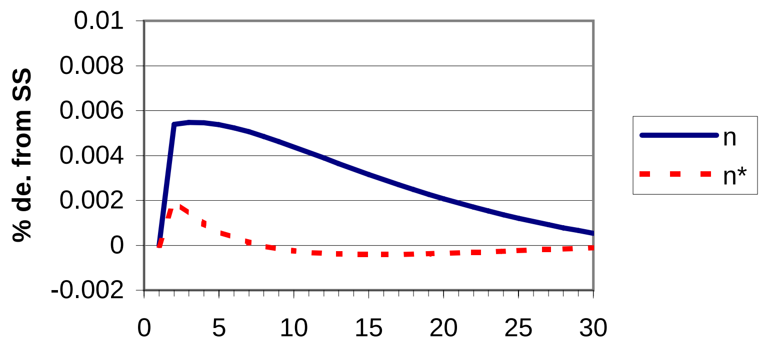


Figure 1B: Capital Utilization Model

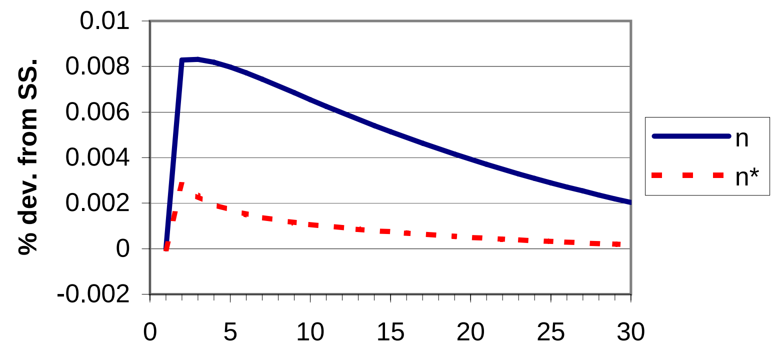


Figure 1C: LBD model

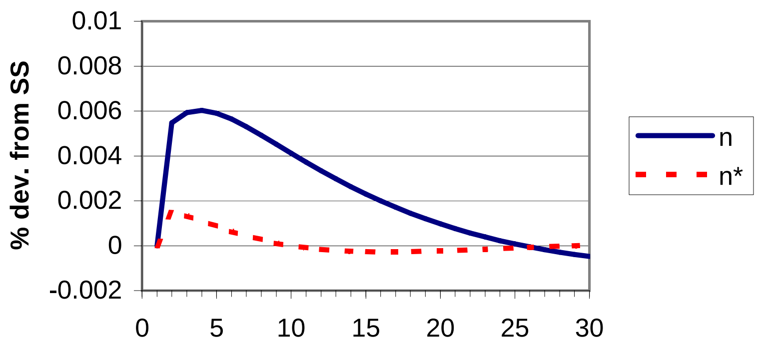


Figure 1D: Full Model

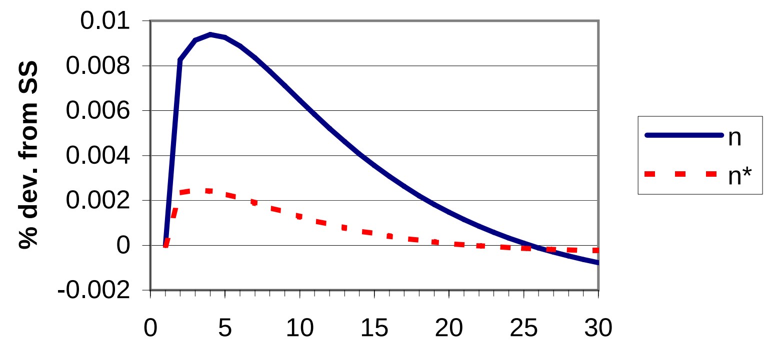
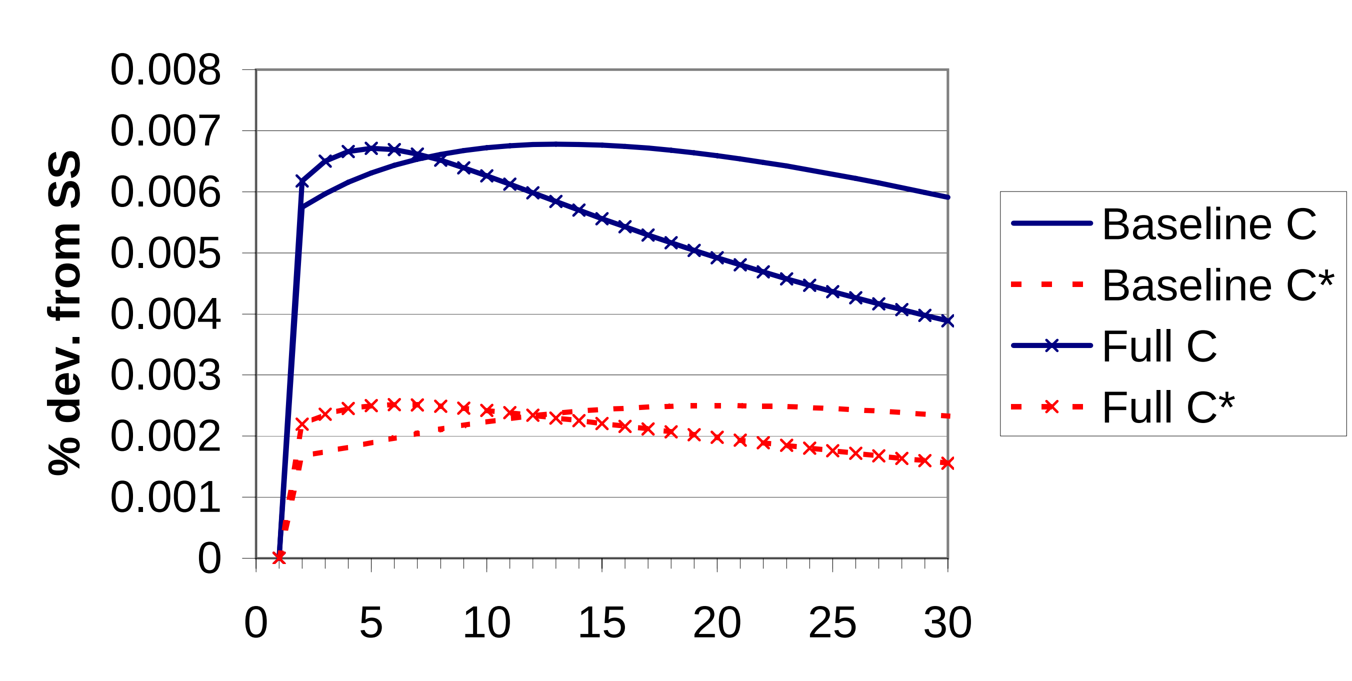


Figure 2: Consumption



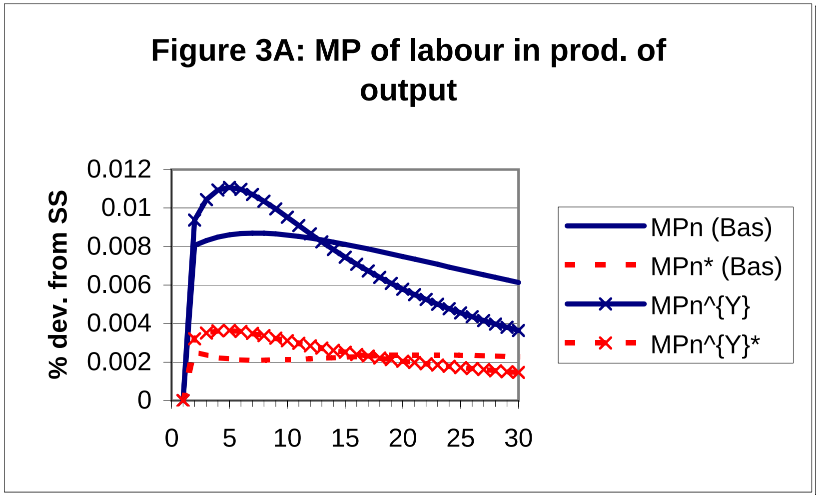


Figure 3A: MP of labour in prod. of output
% dev. from SS
0.012
0.01
0.008
0.006
0.004
0.002
0
0 5 10 15 20 25 30
MPn (Bas)
MPn* (Bas)
MPn^{Y}
MPn^{Y}*

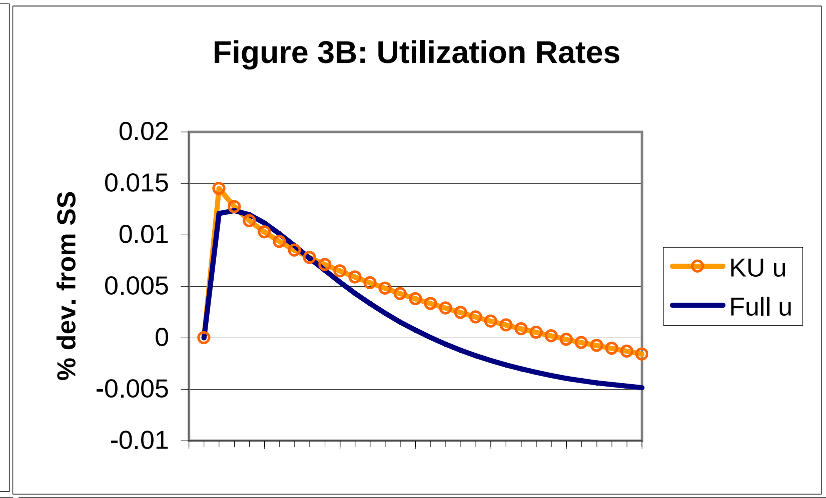


Figure 3B: Utilization Rates
% dev. from SS
0.02
0.015
0.01
0.005
0
-0.005
-0.01
KU u
Full u

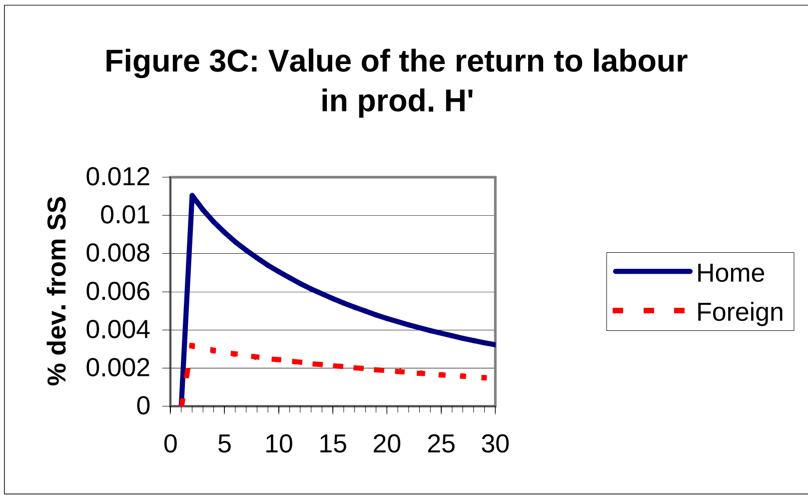


Figure 3C: Value of the return to labour in prod. H'
% dev. from SS
0.012
0.01
0.008
0.006
0.004
0.002
0
0 5 10 15 20 25 30
Home
Foreign

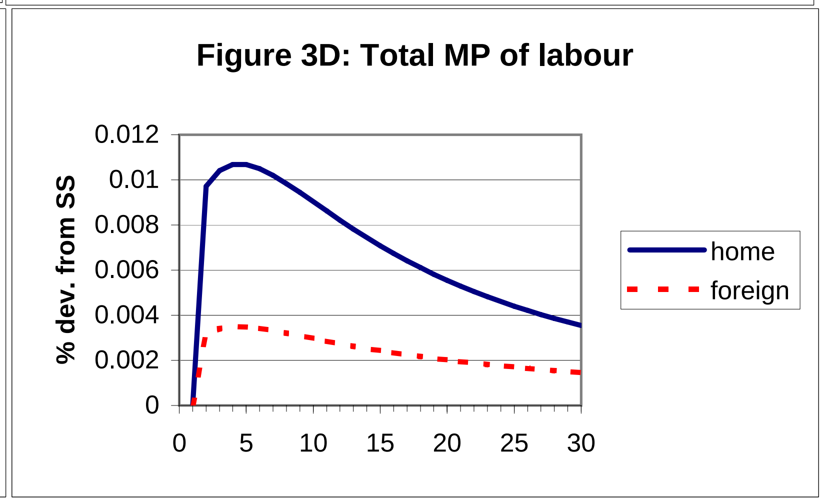


Figure 3D: Total MP of labour
% dev. from SS
0.012
0.01
0.008
0.006
0.004
0.002
0
0 5 10 15 20 25 30
home
foreign

Figure 4A: Baseline model

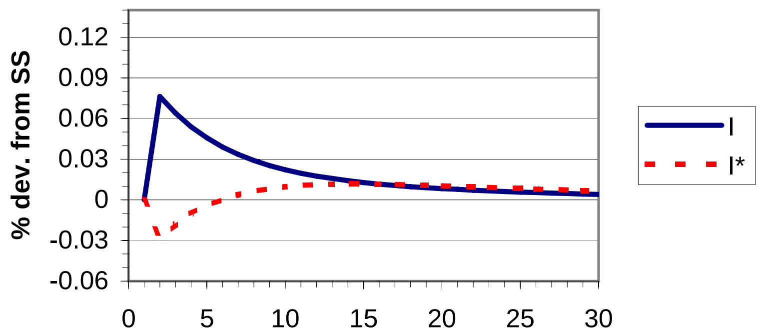


Figure 4B: Capital Utilization Model

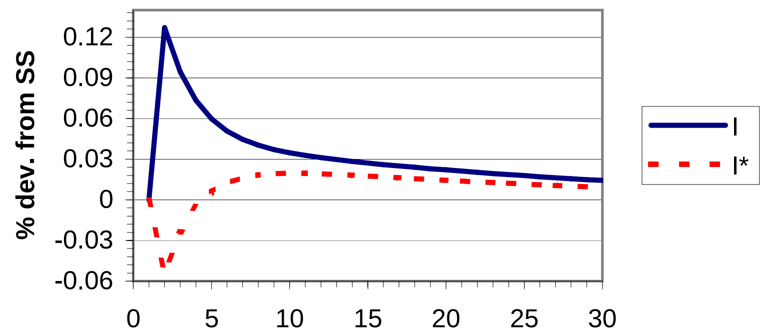


Figure 4C: LBD Model

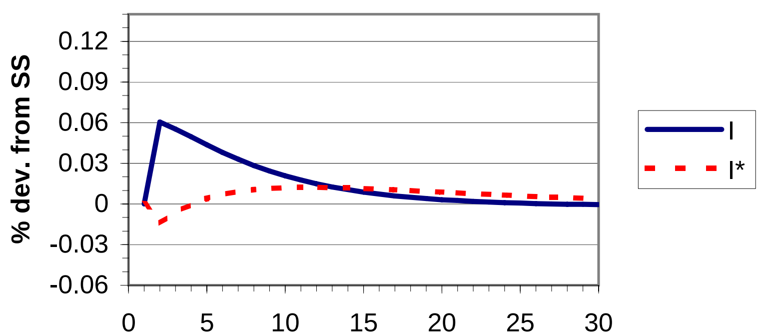


Figure 4D: Full Model

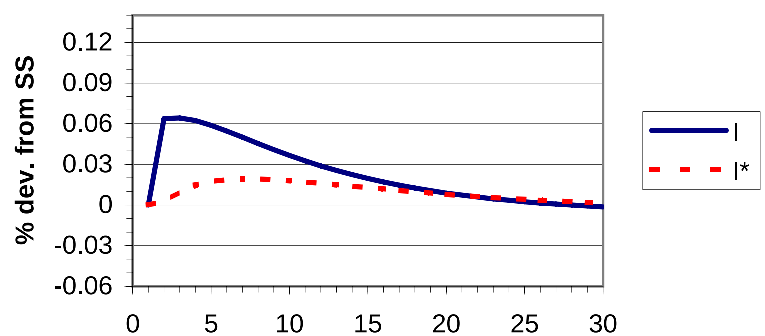


Figure 5A: Realized Interest Rate

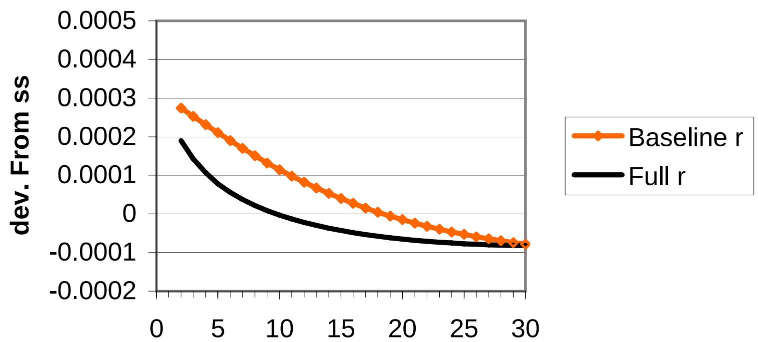
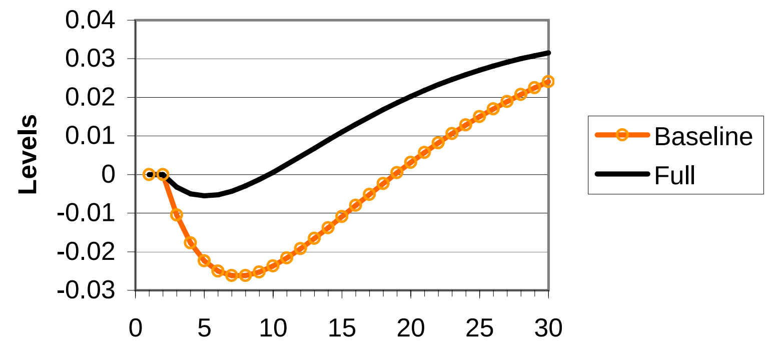
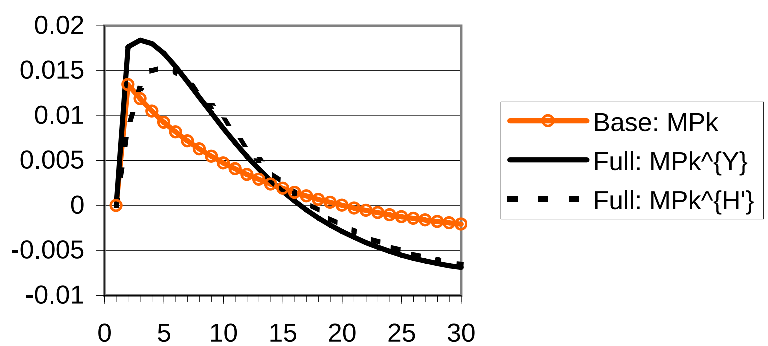


Figure 5B: HC Net Foreign Assets



5C: Marginal Product of K



5D: Marginal Product of K*