Delivering Endogenous Inertia in Prices and Output*

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Abstract

This paper presents a DGE model in which aggregate price level inertia is generated endogenously by the optimizing behaviour of price-setting firms. All the usual sources of inertia are absent here ie., all firms are simultaneously free to change their price once every period and face no adjustment costs in doing so. Despite this, the model generates persistent movements in aggregate output and inflation in response to a nominal shock. Two modifications of a standard one-quarter pre-set pricing model deliver these results: learning-by-doing and habit formation in leisure. While the model delivers persistence, simulations based on estimated shocks to tfp and money growth suggest both output and inflation are too volatile relative to the data and fail to closely follow the historical time series.

Key words: Endogenous price stickiness, Business Cycles, Inflation, Nominal rigidities, Learning-by-doing, Habit formation, Propagation mechanisms, Persistence.

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1 Introduction

There has been a recent surge in interest in dynamic general equilibrium models in which firms adjust their prices infrequently. Many of these models employ one of two classes of time-dependent pricing rules associated with Taylor (1999) and Calvo (1983).[1] While these models have had some success in generating empirically plausible business cycles in response to monetary shocks the pricing arrangements embedded in the models are theoretically unappealing.[2] This theoretical weakness arises from the exogenous nature of the pricing arrangements imposed upon firms which determine both the length of time for which prices cannot be re-optimized as well as the degree of synchronization among firms. This can have important consequences for the ability of these models to predict the response of the economy to changes in the economic environment, especially to changes in monetary policy. While one might expect that the optimal pricing arrangements of firms may respond to policy, they cannot in the model. Since the duration of price stickiness and the degree of staggering of pricing decisions influence the response of aggregate variables in the model, one may not end up with sensible predictions regarding how the economy will respond to these changes.

Staggered price setting models were popular despite this well understood weakness because staggering was viewed as a critical element, along with long periods of price stickiness (for which there was empirical evidence), in generating an inertial response of the price level and aggregate output to monetary shocks. However recent work has questioned the centrality of these two phenomena in propagating nominal shocks. Chari, Kehoe and McGrattan (2000) forcefully argue that staggering of pricing decisions is ineffective in propagating output beyond the assumed duration for which prices are fixed. In addition, Christiano, Eichenbaum and Evans (2005) show that price staggering is not crucial to generating realistic impulse responses. Finally Bils and Klenow (2004) show that on average prices change much more quickly in US data than has been assumed in the sticky price literature.[3]

[1] There are too many sticky price dynamic general equilibrium models to list here. Some examples are King and Watson (1996), Yun (1996), Cho et al (1997), Chari et al. (2000), Bergin and Feenstra (2000), Erceg et al. (2000) and Huang and Liu (2001).

[2] Christiano, Eichenbaum and Evans (2005) show that their model fits the empirical impulse responses quite well.

[3] Bils and Klenow suggest that half of the prices studied lasted no more than 4.3 months which is much shorter than the assumed duration of price rigidity (typically about 12

The goal of this paper is to show that it is not necessary to retain either unappealing element in order to generate inertia not only in output but also in the aggregate price level. To make this point forcefully, the paper restricts the amount of exogenous price rigidity to one period, i.e., all firms set prices simultaneously at the beginning of each period. Nonetheless, the aggregate price level will adjust slowly to a money growth shock because all firms optimally choose to adjust their prices slowly[4]. In the standard one-period sticky price model, firms wish to adjust prices in proportion with the expected change in marginal costs next period. One way to slow down the adjustment of prices is to introduce mechanisms that prevent marginal costs from rising too fast. The other way is for firms to actually choose to adjust prices *less* than proportionally to expected marginal cost. In other words, firms must change their markup. This paper incorporates both of the above features.[5] As a result the model can generate prices that adjust very slowly. Since the firm could in principle adjust fully to expected future inflation after the first period, any subsequent sluggishness seen in the response of prices is endogenous. Indeed, in the standard one period sticky price model, almost all of the adjustment in a firm's price occurs in the first period after the shock.

The paper uses two mechanisms that build upon each other to quantitatively generate realistic persistence in inflation and output. The first mechanism modifies the technological environment in which firms operate by introducing learning-by-doing.[6] While this mechanism generates considerably more inertia than the benchmark one period sticky price model, it does not go far enough. As a result I explore a second mechanism which modifies consumers preferences by introducing habit formation in leisure.[7] I discuss them in turn.

Learning-by-doing influences price inertia in two ways by providing a

months) in the literature.

[4]This is not induced by imposing menu costs on the firms.

[5]Some recent papers that try to generate price inertia by dampening the response of marginal cost are Altig et al (2005), Neiss and Pappa (2005) and Danthine and Kurmann (2004). Huang (2006) points out that some of these mechanisms may offset each other and impede the propagation of monetary shocks. Bils and Chang (2000) provide some other interesting models that generate markup variation.

[6]See Arrow (1962) and especially Rosen (1972) for early discussions of learning-by-doing as a by-product of production experience.

[7]In contrast to habit formation in leisure, it is quite common to use habit formation in consumption in sticky price DGE models. An early example is Smets and Wouters (2003).

dynamic link between current production and future productivity. The basic mechanism is quite intuitive. As firms raise output to meet the increase in demand that follows an expansion of the money supply, they accumulate production knowledge which lowers future costs. In the periods after the shock, when firms are free to set prices, they face lower marginal costs and thus set lower prices as compared to an environment without learning-by-doing. In addition to this, a more subtle mechanism may be in operation - firms may actually choose to lower their markup over marginal cost. This occurs because firms face a trade-off between increasing current revenue and reducing future marginal costs. As a result, firms use prices to control how much they learn in any given period depending on the marginal value of that learning to the firm. To see this, consider a firm that desires to reduce future costs via learning-by-doing. To do this, it must increase output. Given demand, in a monopolistically competitive environment, the firm must lower its price in order to sell this extra output.[8] This has two implications for the model. First, firms will set lower prices compared to standard models in which this dynamic trade-off is absent. In other words the steady state markup charged by firms is lower. Second, this markup will respond to shocks that shift the demand for the firm's product, such as a money growth shock.

Consider an increase in the growth rate of money which leads to an increase in the demand for a firm's product. This creates a favorable environment for learning because the demand curve is more responsive to a cut in price than in steady state. Essentially, a unit reduction in price yields more learning bang for the buck by generating more production and greater future cost reductions as compared to a similar price reduction in steady state. If the marginal value of learning is high for a firm, this mechanism makes it want to lower the markup it charges over marginal cost relative to steady state.[9] When combined with the reductions in marginal cost induced by learning-by-doing, the lower markups can be a potent mechanism for generating price inertia.

Learning-by-doing also acts as a real propagation mechanism. The short-lived increases in output generated by a standard sticky price model in response to an unexpected increase in the growth rate of money are converted

[8] I am abstracting from the possibility of inventories. Obviously a firm could, for a limited amount of time, produce output and store it rather than reduce price now. Given storage costs, this output must eventually be sold, lowering prices at that time.

[9] The converse is also true. If the marginal value of learning is low (perhaps because of high production in the recent past), the markup may be raised.

into long-lived increases due to the fact that productivity is above steady state for a number of periods.

The other mechanism that contributes to the inertial response of the model is the presence of habit formation in the utility function with regards to the desire for leisure. Habits in leisure imply that consumer utility today depends not only on the current level of leisure but also on past levels. Due to habit formation, high levels of leisure in the past lead to an increased desire for leisure in the present, other things being constant. To see how this might generate an inertial response in hours and output, consider the response of hours in a sticky price model in the absence of habit formation. A one time fall in the growth rate of money leads to a reduced demand for output, leading to a sharp increase in leisure as firms cut production in the impact period. The next period, prices fall and money balances are restored almost to steady state levels. As a result, output, hours and leisure revert to virtually steady state levels. When consumers form habits, the desire for leisure rises as reflected in a rise in the marginal utility of leisure in the second period. As a result, they are reluctant to return immediately to steady state levels of leisure and accordingly hours stay below steady state for a number of periods. This inertial response of hours also generates more output persistence which in turn leads to inflation persistence.

The effectiveness of these two mechanisms in generating quantitatively realistic levels of inertia in output and the aggregate price level is evaluated in the context of a dynamic general equilibrium model with real money balances in preferences. Prices, in this model, are pre-set for one period only. Simulations from a linearized version of the model, calibrated to the US economy, show that the model is capable of generating inertia in the aggregate price level as well as in output that is close to that observed in the aggregate US data. The first order auto-correlation coefficient of detrended output and inflation in US data is .97 and .88 respectively. When the benchmark sticky price model with neither habits nor learning-by-doing is subject to technology and money shocks it generates .73 and .07 respectively for the same moment. The full model with both mechanisms built in delivers .94 and .88 respectively. The model also delivers realistic values for the usual second moments discussed in the business cycle literature.

Despite this success, some shortcomings remain, especially in the ability of the model to capture the dynamics of inflation. Plots of simulated inflation and output based on estimated shocks to the solow residual and to the growth rate of M2 for the US economy reveal significant departures from the historical data. While it is clear that the full model improves the fit of inflation relative to the sticky price benchmark by increasing persistence and lowering volatility, simulated inflation remains too volatile, as does output.

As far as I am aware there are no previous studies of closed economy business cycles that incorporate learning-by-doing into monetary dynamic general equilibrium models.[10] However, Cooper and Johri (2002), show that learning-by-doing is extremely effective at propagating technology shocks in a real business cycle framework. Cooper and Johri use a representative agent framework and are agnostic about the issue of who actually learns from past production: workers or firms, and thus offer no account of possible decentralizations. In complementary work, Chang, Gomes and Schorfheide (2002) focus solely on learning that is embodied in workers and is fully captured in wages. They estimate the aggregate learning rate for the US using data from the PSID and incorporate this into a dynamic general equilibrium model with real shocks. They too are able to generate a persistent response of output to real shocks.

There is similarly little work on habit formation in leisure in the context of dynamic general equilibrium models with money shocks. An example is Yun (1996).[11] A few more studies can be found in the real business cycle literature. Two such studies are Bouakez and Kano (2006) and Wen (1998). Unlike Yun, in both papers, the stock of habit is formed based on a long lived distributed lag over past levels of leisure (or hours). My paper differs from Bouakez and Kano in a number of ways. First, agent's habits are based on leisure in the previous quarter only whereas it involves an infinite distributed lag in their paper. Second, they solve a central planner's problem without the inherent distortions associated with imperfect competition and sticky prices. Third, the source of shocks in their paper are non-monetary. The main goal of their paper is to show that the infinite lag in leisure implied by habit formation is akin to the worker based learning model of Chang et al (2002). This near equivalence does not hold in my model. As discussed above, the

[10] The closest models, Tsuruga (2007) and Cook(1999), incorporate dynamic production externalities to propagate monetary shocks. Cooper and Johri (1997) discuss how these externalities may be interpreted as learning effects. Unlike the current model, these dynamic externalities do not generate endogenous markup movements. While potentially very important, externalities are ignored in this paper to focus on the impact of internal learning-by-doing on pricing decisions.

[11] It is also quite common since the work of Smets and Wouters (2003) to incorporate habit formation in consumption into monetary DGE models.

main goal of introducing habit formation in leisure is to temper the spikes in hours worked implied by sticky price models in response to monetary shocks.

The next section presents the model and discusses the two mechanisms in more detail. Section 3 presents some analytical results for a simplified economy and compares it to some well known sticky price models. Section 4 discusses the calibration of key parameters, section 5 presents simulation results. The final section concludes.

2 The Model

This section describes a monetary economy populated by many identical, infinitely lived consumers. Each period, the economy finds itself in one of finitely many states, s_t. Let $s^t = (s_{0,\ldots,}s_t)$ be the history of these states of the world. Along with labour and a good that is used both for consumption and investment, the commodities in the economy are money, a continuum of intermediate goods, and organizational capital.

2.1 Firms

There are a large number of final good producers who behave competitively and use the following technology for converting intermediate goods indexed by $i \in [0, 1]$ into final goods.

$$Y(s^t) = [\int Y_i^{\eta}(s^t)di]^{\frac{1}{\eta}} \tag{1}$$

Each period they choose inputs $Y_i(s^t)$ for all $i \in [0, 1]$, and output $Y(s^t)$ to maximize profits given by

$$\max P(s^t)Y(s^t) - \int P_i(s^{t-1})Y_i(s^t)di \tag{2}$$

subject to (1) where $P(s^t)$ denotes the price of the final good at history s^t,while $P_i(s^{t-1})$ is the price paid for the ith intermediate good in period t. Note that these prices are set before the realization of the period t shock. The solution to this problem gives us the input demand functions:

$$Y_i^d(s^t) = (\frac{P(s^t)}{P_i(s^{t-1})})^{\frac{1}{1-\eta}}Y(s^t). \tag{3}$$

The zero profit condition can be used to infer the level of final goods prices from the intermediate good prices:

$$P(s^t) = [\int P_i^{\frac{\eta}{\eta-1}}(s^{t-1})di]^{\frac{\eta-1}{\eta}}. \tag{4}$$

There are a large number of intermediate goods producers, indexed by the letter i that operate in a Dixit-Stiglitz style imperfectly competitive economy. Each of these produces intermediate goods with a technology given by F(.) which is increasing in all inputs :

$$Y_i(s^t) = F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)). \tag{5}$$

Here $N_i(s^t)$ is the amount of labor hired, $K_i(s^t)$ is the amount of physical capital hired by the firm to produce output, $Y_i(s^t)$, and $A(s^t)$ is a common term governing the level of total factor productivity. In addition to these conventional inputs, the firm carries a stock of organizational capital, $H_i(s^{t-1})$, which is an input in the production technology. Organizational capital refers to the information accumulated by the firm in the process of past production regarding how best to organize its production activities and deploy its inputs.[12] As a result, the higher the level of organizational capital, the more productive the firm. Learning-by-doing leads to the accumulation of organizational capital which depends on output and the current stock of organizational capital:

$$H_i(s^t) = H_i^{\gamma}(s^{t-1})Y_i^{\phi}(s^t). \tag{6}$$

All producers begin life with a positive and identical endowment of organizational capital. I assume $0 < \gamma < 1$and $0 < \phi \leq 1$.

While learning-by-doing is often associated with workers and modeled as the accumulation of human capital, a number of economists have argued that firms are also store-houses of knowledge. Atkeson & Kehoe (2005) note "At least as far back as Marshall (1930, bk.iv, chap. 13.I), economists have argued that organizations store and accumulate knowledge that affects their technology of production. This accumulated knowledge is a type of unmeasured

[12] Atkeson and Kehoe (2005) model and estimate the size of organizational capital for the US manufacturing sector and find that it has a value of roughly 66 percent of physical capital.

capital distinct from the concepts of physical or human capital in the standard growth model." Similarly Lev & Radhakrishnan (2003) write, "Organization capital is thus an agglomeration of technologies—business practices, processes and designs, including incentive and compensation systems—that enable some firms to consistently extract out of a given level of resources a higher level of product and at lower cost than other firms." The notion of organizational capital encompasses Rosen's (1972) view of it as a firm specific capital good as well as knowledge embodied in the matches between workers and tasks within the firm.

This specification of how learning-by-doing leads to productivity increases draws on early work by Arrow(1962) and Rosen(1972) as well as a large empirical literature dating back roughly a hundred years which documents the pervasive presence of learning effects in virtually every area of the economy. Recent studies include Bahk & Gort (1993), Irwin & Klenow (1994), Jarmin (1994), Benkard (2000), Thompson (2001), Thornton & Thompson (2001) and Cooper & Johri (2002). The current specification is taken from Cooper and Johri (2002) which not only offers a detailed justification for the modelling assumptions but also a number of estimates of the learning technology at different levels of aggregation for the US economy.

Briefly, the specification allows for the sensible idea that production knowledge may become less and less relevant over time as new techniques of production, new product lines and new markets emerge. Second, it allows in a general way for the idea that some match specific knowledge may be lost to the firm as workers leave or get reassigned to new tasks or teams within the firm. In addition, the knowledge accumulated through production experience will be a function of the current vintage of physical capital. The decision to replace physical capital will imply that the existing stock of organizational capital will be less relevant. If we think of the internal context of firms as an environment with an ever changing set of tasks, workers, teams, machines and information then it makes sense to model organizational capital as continually accumulating and depreciating much as we do with physical capital.

The restriction $\gamma < 1$ is consistent with the empirical evidence supporting the hypothesis of depreciation of organizational capital often referred to as organizational forgetting. Argote et al. (1990) provide empirical evidence for this hypothesis of organizational forgetting associated with the construction of Liberty Ships during World War II. Similarly, Darr et al. (1995) provide evidence for this hypothesis for pizza franchises and Benkard (2000) provides evidence for organizational forgetting associated with the production of commercial aircraft. One difference between these studies and this paper is that the accumulation technology is log-linear rather than linear. Clarke (2006) shows that the additional curvature in this log-linear technology is unlikely to produce predictions for aggregate variables, in response to a technology shock, considerably different to those associated with a linear technology. It is the implied dynamic structure associated with the accumulation of organizational capital, rather than any functional form assumptions that drives the results in Cooper and Johri (2002). Similar results should follow in the current context.

Each intermediate goods producer faces a downward sloping demand function for his product (3) which comes from the profit maximization problem of the final goods producers discussed above. Prices are set by all producers at the beginning of each period before the realization of the state s_t and cannot be changed during the period once set. Thus there are two differences between the intermediate goods firm's problem in the typical staggered price-setting model and this paper. First, the technology has been modified to incorporate learning-by-doing. Second, firms set prices for one period in a synchronized way which is a special case of an N period overlapping contracts structure with N=1.

At the beginning period t, each producer chooses a price $P_i(s^{t-1})$ before the shocks are realized and the level of organizational capital, $H_i(s^t)$, after the shock is realized to maximize discounted profits:

$$\max \sum_{t=1}^{\infty} \sum_{s_t} Q(s_t)[P_i(s^{t-1})Y_i(s^t) - P(s^t)V(s^t)]$$

subject to (3) and (6) where $Q(s_t)$ is the appropriate discount rate to use to price revenue and costs in the initial period which is determined in the household problem. $V(s^t)$ denotes the real cost function which has the real wage, $w(s^t)$,the real rental rate on capital, $r(s^t), H_i(s^{t-1})$ and $Y_i(s^t)$ as arguments. The cost function is obtained from the cost minimization exercise:

$$V_i(s^t) = \min_{N_i, K_i} w(s^t)N_i(s^t) + r(s^t)K_i(s^t) \tag{7}$$

subject to

$$F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)) \geq \overline{Y}. \tag{8}$$

The solution to this minimization problem implies

$$\frac{w(s^t)}{r(s^t)} = \frac{F_N(s^t)}{F_K(s^t)} \tag{9}$$

and

$$F(K_i(s^t), N_i(s^t), H_i(s^{t-1}), A(s^t)) = \overline{Y} \tag{10}$$

from which the input demands can be obtained. Substituting these into (7) yields the cost function. Taking $V_i(s^t)$ as given, the solution to the maximization problem above implies two first order conditions:

$$\lambda^F(s^t) = \tag{11}$$
$$\sum_{s^{t+1}} Q(s^{t+1} \mid s^t) \left\{ \lambda^F(s^{t+1}) \Phi'_H(H_i(s^t), Y_i(s^{t+1})) - P(s^{t+1}) V'_{Hi}(H(s^t), s^{t+1}) \right\}$$

and

$$\sum_{s^t} Q_t[Y_{it}(\frac{-\eta}{1-\eta}) + P_t V'_{Yit}(\frac{1}{1-\eta}) \frac{Y_{it}}{P_{it}(s^{t-1})} -$$
$$\lambda_t^F \Phi'_Y(H_{it}, Y_{it})(\frac{1}{1-\eta}) \frac{Y_{it}}{P_{it}(s^{t-1})}] = 0 \tag{12}$$

where $\lambda^F(s^t)$ is the Lagrange multiplier associated with the organizational capital accumulation equation once (3) has been used to substitute out for $Y_i(s^t)$. The latter first order condition determines the optimal level of prices to be set by the producer. Note that the state has been suppressed in this equation except where it is needed to avoid confusion. Raising prices by one unit causes output to fall since producers face downward sloping demand curves for their product. The first term captures the net impact on current revenue of the higher price but lower output, while the second term represents the current cost savings from producing less. The third term appears because the producer realizes that he faces a forward looking problem due to learning-by-doing. The accumulation equation for organizational capital (6) implies that a reduction of current period output will lead to a reduction in organizational capital available tomorrow. The third term captures the value of this organizational capital lost to the firm and is made up of three parts. The term $(\frac{1}{1-\eta})\frac{Y_{it}}{P_{it}}$ represents the reduction in output due to the higher price, while $\Phi'_Y(H_{it}, Y_{it})$ represents the reduction in H_{it+1}due to the reduction in output which must be evaluated at λ_t^F, the marginal value of organizational capital to the firm.

Equation (11) determines the value of having available an additional unit of organizational capital for use by the firm in the following period. First, the additional organizational capital improves profits by reducing costs, as captured by the second term on the right hand side (recall V'_{Hit+1} is negative). Second, it adds to the ability of the organization to learn from production thus raising future organizational capital. This additional organizational capital has a value of λ_{t+1}^F for the firm. All this must be discounted by the price of one dollar in period t+1 in units of period t dollars. Alternatively, one could say that the firm sets prices so that the value of accumulating an additional unit of organizational capital today is just equal to the discounted value of organizational capital tomorrow.

The intuition in (11) and (12) suggests that firms face a trade-off between current profits and future profits which is not present in the traditional price setting problem. Charging a higher price today lowers the amount of organizational capital available tomorrow which raises future costs and lowers future profits. As a result, firms will optimally select a lower price in the presence of learning by doing than they would otherwise set. This can be seen by re-writing (12) as

$$P_{it}(s^{t-1}) = \frac{\sum_{s^t} Q_t Y_{it}(P_t V'_{Yit} - \lambda_t^F \Phi'_{Yit})}{\eta \sum_{s^t} Q_t Y_{it}} \tag{13}$$

and noting that the second term appears only when the learning-by-doing mechanism is present.

Note that each intermediate good firm earns positive profits even in the presence of a constant returns to scale technology due to the accumulation of organizational capital. However there is no entry or exit in this industry by assumption.

2.2 Consumers

The economy is populated by a large number of identical consumers whose preferences are defined over consumption of final goods ($C(s^t)$), leisure ($L(s^t)$)

and real money balances $(M(s^t)/P(s^t))$. The preference specification below allows for endogenous habit formation with $b \geq 0$ being the parameter which determines the degree of habit persistence. These preferences reduce to the standard specification with no habits for $b = 0$. The benchmark specification will have no habits so that $b = 0$. Each consumer maximizes the sum of discounted expected utility subject to a sequence of budget constraints (given below) by choosing the optimal quantity of these goods to consume, the amount of hours to work and how much to invest $x(s^t)$in physical capital $(K(s^t))$ and one-period nominal bonds $B(s^{t+1})$ each period. They take as given prices $(P(s^t))$, wages $(w(s^t)$) and interest rates $(r(s^t)$). If $0 < \beta < 1$ is the discount factor, then the household's problem is to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) U\left(C(s^t), L(s^t) - bL(s^{t-1}), \frac{M(s^t)}{P(s^t)}\right), \tag{14}$$

subject to the sequence of budget constraints:

$$\begin{aligned} & P(s^t)C(s^t) + M(s^t) - M(s^{t-1}) + \sum_{s_{t+1}} Q(s^{t+1} | s^t)B(s^{t+1}) + P(s^t)x(s^t) \\ \leq \; & P(s^t)\left[w(s^t)N(s^t) + r(s^t)K(s^{t-1}) + T(s^t)\right] + B(s^t) + \Pi(s^t), \end{aligned} \tag{15}$$

and

$$N(s^t) + L(s^t) \leq 1. \tag{16}$$

Consumers lend out their stock of physical capital and labor services to intermediate goods producers and receive wages and interest income. Each of the nominal bonds, $B(s^{t+1})$, provides one dollar in state s^{t+1}at the expense of $Q(s^{t+1} \mid s^t)$ dollars in state s^t. In addition, as owners of all firms, they receive $\Pi(s^t)$ which is the current profits of intermediate goods producers and $T(s^t)$, the current real net transfers from the monetary authority. The initial conditions $K(s^{-1}), M(s^{-1}), B(s^0)$ are also given. Consumers face quadratic costs of adjusting the capital stock. In particular, the capital stock evolves according to

$$K(s^t) - (1-\delta)\, K\left(s^{t-1}\right) = x(s^t) - \frac{v}{2}\left(\frac{x(s^t)}{K(s^{t-1})} - \delta\right)^2 K(s^{t-1}) \tag{17}$$

While these adjustment costs are a real resource cost for the economy, since investment equals depreciation in steady state, no adjustment costs are incurred in steady state.

In addition to the above constraints, we need the sequence of borrowing constraints $B(s^{t+1}) \geq B^u$ for some large negative value of B^u.

The first-order conditions for optimality are given by

$$U_L(s^t) = U_C(s^t)w(s^t) + \beta b \sum_{s_{t+1}} \pi(s^{t+1} | s^t) U_L(s^{t+1}) \quad (18)$$

$$\frac{U_m(s^t)}{P(s^t)} - \frac{U_C(s^t)}{P(s^t)} = -\beta \sum_{s_{t+1}} \pi(s^{t+1} | s^t) \frac{U_C(s^{t+1})}{P(s^{t+1})} \quad (19)$$

$$Q(s^{t+1} | s^t) = \beta \pi(s^{t+1} | s^t) \frac{U_C(s^{t+1})}{U_C(s^t)} \frac{P(s^t)}{P(s^{t+1})} \quad (20)$$

$$\frac{U_C(s^t)}{1 - v\left(\frac{x(s^t)}{K(s^{t-1})} - \delta\right)} = \beta \sum_{s_{t+1}} \pi(s^{t+1} | s^t) \frac{U_C(s^{t+1})}{1 - v\left(\frac{x(s^{t+1})}{K(s^t)} - \delta\right)} D(s^{t+1}) \quad (21)$$

where $U_j(s^i)$ denotes the derivative of U with respect to variable j evaluated in state s^i. Note that

$$D(s^{t+1}) = r(s^{t+1}) \left[1 - v\left(\frac{x(s^{t+1})}{K(s^t)} - \delta\right)\right] + 1 - \delta + \frac{v}{2}\left[\left(\frac{x(s^{t+1})}{K(s^t)}\right)^2 - \delta^2\right]$$

The interpretation of these first order conditions is quite standard. Equation (18) gives the optimal labor-leisure choice. The presence of habits adds a second term on the right hand side and introduces dynamics into the decision. An extra unit of leisure today generates not only some positive current marginal utility but it also raises the desire for leisure tomorrow by increasing the marginal utility of leisure tomorrow. (19) is the optimality condition determining money demand. It states that the consumer should choose to save nominal balances to the point that the current net benefit of saving an additional dollar (which is made up of the marginal utility lost due to lower current consumption minus the marginal utility gained due to higher money balances) is just equal to the discounted expected benefit next period(composed of the marginal utility of the extra consumption that can be bought next period which in turn depends on the expected value of inflation over this interval). Equation (20) is the equation which determines optimal bond holdings while equation (21) is the optimality condition for capital accumulation. This condition looks slightly different from the standard intertemporal euler equation due to the presence of adjustment costs. Since $v \geq 0$, at the optimum, the household endogeneizes the fact that one unit of foregone consumption today produces only $1 - \upsilon\left(\frac{x(s^t)}{K(s^{t-1})} - \delta\right)$ units of capital tomorrow. Also, note that $\pi(s^{t+1}|s^t) = \pi(s^{t+1})/\pi(s^t)$ is the probability of s^{t+1} conditional on s^t having been realized.

The nominal money supply process is

$$M(s^t) = \mu(s^t)M(s^{t-1}) \tag{22}$$

where $\mu(s^t)$ is a stochastic process. Consumers receive lump sum transfers of new money balances which satisfy:

$$P(s^t)T(s^t) = M(s^t) - M(s^{t-1}). \tag{23}$$

In addition to these first order conditions from the consumer and firm problem we have market clearing conditions which require that the total stock of capital supplied by consumers is equal to the sum of capital rented by all intermediate goods firms. Similarly the total hours of labor supplied by consumers should equal the sum of labor hours demanded by all intermediate goods firms. Recall that while prices are chosen by firms before uncertainty about shocks is resolved, factor demands are chosen afterwards. Bond market clearing requires that $B(s^{t+1}) = 0$. The resource constraint for the economy is

$$C(s^t) + x(s^t) = Y(s^t). \tag{24}$$

An equilibrium is a collection of allocations for consumers, $C(s^t), N(s^t)$, $x(s^t)$, $B(s^{t+1})$ and $M(s^t)$; allocations for intermediate goods firms: $N_i(s^t)$, $K_i(s^t), H_i(s^{t-1})$ for all i $\in [0,1]$; allocations for final goods firms: $Y(s^t), Y_i^d(s^t)$ for all i $\in [0,1]$; together with prices $w(s^t), r(s^t), Q(s^{t+1} \mid s^t), P(s^t), P_i(s^{t-1})$, for all i $\in [0,1]$ that satisfy the following conditions: i) taking prices as given the consumer allocations solve the consumer's problem; ii) taking all prices but its own as given, each intermediate goods producer's price and stock of organizational capital satisfies (12) and (13); iii) taking prices as given the final goods producers allocations solve the final goods producer problem; iv) the factor market conditions and resource constraint hold and the bond market clears. Only symmetric equilibria in which all consumers and producers behave identically are studied.

3 Some Analytical Results

In order to gain some additional insight into the propagation mechanisms being discussed in the paper, this section presents some analytical results regarding the dynamics of prices and output. I will refer to the model with only learning effects as the *lbd model* and the model with habit formation in leisure as the *habits model*. These are compared to each other and to two well known models of the sticky price literature. The first model is our benchmark in which prices are pre-set for one period will be referred to as the *preset pricing model*. The second model has prices set for two periods with staggering and was discussed extensively in Chari et al (2000). This will be called the *staggered pricing model*. As is usual, I make some simplifications in order to solve the model analytically. I assume that there is no physical capital in the economy and impose a static money demand equation instead of (19) which implies $M/P = Y$. I will also assume that $\alpha = \beta = 1$ and separability of leisure in preferences. For the models without habits, preferences may be specified as

$$U(C, L, M/P) = \tfrac{1}{1-\sigma}[(\omega C^{(\theta-1)/\theta} + (1-\omega)(\frac{M}{P})^{(\theta-1)/\theta})^{\frac{\theta}{\theta-1}}]^{1-\sigma} + \frac{1}{1-\psi}L^{1-\psi}. \tag{25}$$

In the presence of habit formation L_t is replaced with $L_t^* = L_t - bL_{t-1}$ where b governs the extent to which habits are formed.

3.1 The models without habits or learning

I begin with the equations that govern price setting behaviour in the two comparison models. In the preset pricing model,

$$P_i(s^{t-1}) = \frac{(1/\alpha)\sum_{s^t} Q(s^t \mid s^{t-1})P(s^t)w(s^t)N_i(s^t)}{\eta \sum_{s^t} Q(s^t \mid s^{t-1})Y_i(s^t)}. \tag{26}$$

Substituting for Q from (20) and linearizing leads to

$$\widehat{P_{it}} = E(\widehat{P}_t + \widehat{w}_t). \tag{27}$$

The equivalent expression for the staggered pricing model is available from equation (27) of Chari et al (2000) :

$$\widehat{P_{it}} = (\frac{1}{2})E(\widehat{P}_t + \widehat{w}_t + \widehat{P_{t+1}} + \widehat{w_{t+1}}). \tag{28}$$

Note that the pricing decision in the pre-set pricing model depends only on current period variables. The expectations operator occurs because prices must be chosen in advance of the realization of money shocks. By contrast the staggered pricing model involves forward looking variables that have the potential to create some dynamics. Our next step is to obtain an expression for $\widehat{w_t}$ in order to substitute it out. The static money demand equation implies that $\widehat{M_t} - \widehat{P_t} = \widehat{C_t} = \widehat{Y_t}$. Given separability of preferences, we can use (18) and market clearing conditions to obtain the following expression: $\widehat{w_t} = \sigma\widehat{Y_t} + \chi\widehat{N_t}$ where $\chi = (N/1-N)\varrho$. In this expression, N denotes steady state hours and ϱ is the elasticity of the marginal utility of leisure w.r.t. leisure. Substituting out for $\widehat{N_t} = \widehat{Y_t}$ we get $\widehat{w_t} = (\sigma+\chi)\widehat{Y_t}$. Since $\sigma \geq 1$ and $\varrho = \sigma$ in our calibration, $\Lambda = \sigma + \chi \geq 1$ as was the case in Chari et al (2000) where $\widehat{w_t} = \Lambda\widehat{Y_t}$.

We can replace the expression for wages into the pricing equation (28) to get

$$\widehat{P_t} = (\frac{1}{2})E(\widehat{P_{t-1}} + \widehat{P_{t+1}}) + \Lambda(\widehat{Y_t} + \widehat{Y_{t+1}}). \tag{29}$$

Replacing $\widehat{Y_t}$ with $\widehat{M_t} - \widehat{P_t}$ and re-arranging gives us

$$E\widehat{P_{t+1}} - 2\frac{1+\Lambda}{1-\Lambda}\widehat{P_t} + \widehat{P_{t-1}} = -2\frac{\Lambda}{1-\Lambda}E(\widehat{M_{t+1}} + E\widehat{M_t}). \tag{30}$$

3.2 The lbd model

Turning next to pricing in the lbd model, recall first that it is built on the framework of the preset pricing model. Therefore any dynamic elements that appear are the result of the learning mechanism that I have introduced.

The key equation that governs the dynamics of prices emerges from the maximization problem of intermediate goods firms which solve the following problem.

$$\max \sum_{t=1}^{\infty} \sum_{s_t} Q(s_t)[P_i(s^{t-1})Y_i(s^t) - P(s^t)w(s^t)N_i(s^t)] \tag{31}$$

subject to (3) and (6). For our purposes, it is useful to substitute the input demand function (3) in (6) and derive an expression for P_i, the input price set by the ith firm:

$$P_i(s^{t-1}) = P(s^t)H_i^{\frac{\gamma(1-\eta)}{\phi}}(s^{t-1})Y^{1-\eta}(s^t)H_i^{\frac{-(1-\eta)}{\phi}}(s^t). \quad (32)$$

Note that the joint implication of a downward sloping demand curve and the accumulation equation for organizational capital is that prices are decreasing in $H(s^t)$ and increasing in $P(s^t), Y(s^t)$ and $H(s^{t-1})$. While I have discussed the reasons why prices must be reduced in order to speed up the accumulation of organizational capital, it is worth explaining why prices are increasing in $H(s^{t-1})$, the current level of organizational capital. The accumulation technology (6) implies that additional units of organizational capital today increase the ability of the firm to learn, leading to more organizational capital tomorrow. As a result of this improved efficiency in learning, less needs to be produced today in order to achieve any target level of organizational capital tomorrow. Given downward sloping demand curves facing firms, this implies that prices can be higher.

Assuming no capital and a Cobb-Douglas production fucntion implies

$$N_i(s^t) = Y_i^{1/\alpha}(s^t)H_i^{-\epsilon/\alpha}(s^{t-1}). \quad (33)$$

Replacing (32) and (33) in (31) and maximizing over $H(s^t)$ yields an efficiency condition for the ith intermediate goods firm which is the equivalent of combining (11) and (12): our dynamic pricing equation. Rearranging yields the following expression:

$$\begin{aligned} &\sum_{s^t} Q(s_t)[\frac{\eta}{\phi}P_i(s^{t-1})\frac{Y_i(s^t)}{H_i(s^t)} - \frac{1}{\phi}P(s^t)w(s^t)H_i^{-\epsilon/\alpha}(s^{t-1})\frac{1}{\alpha}\frac{Y_i^{1/\alpha}(s^t)}{H_i(s^t)}] + \\ &\sum_{s^{t+1}} Q(s_{t+1})\left\{\begin{array}{c} P(s^{t+1})w(s^{t+1})\frac{\epsilon}{\alpha}H_i^{-(\epsilon/\alpha)-1}(s^t)Y_i^{1/\alpha}(s^{t+1}) + \\ \frac{\gamma}{\phi}[-\eta P_{it}(s^t)\frac{Y_i(s^{t+1})}{H_i(s^t)} + \frac{P(s^{t+1})w(s^{t+1})H_i^{-\epsilon/\alpha}(s^t)Y_i^{1/\alpha}(s^{t+1})}{\alpha H_i(s^t)}] \end{array}\right\} \\ = \ & 0 \end{aligned} \quad (34)$$

Using symmetry in equilibrium gives us : $P_i(s^{t-1}) = P(s^t), Y_i(s^t) = Y(s^t), H_i(s^t) = H(s^t)$ for all states. In deriving the linearized version of (34) around the deterministic steady state, we will use $\alpha = \beta = 1$ and $Q(s_{t+1})=Q(s_t)*Q(s^{t+1} \mid s^t)$ to get

$$\begin{aligned} E\{B_1(\widehat{P}_t + \widehat{Y}_t) - E(\widehat{w}_t - \epsilon\widehat{H}_t)\} &= E\{-(\epsilon\phi + \gamma)E(\widehat{w_{t+1}} - \epsilon\widehat{H_{t+1}}) + \\ &\quad B_1(\widehat{Y_{t+1}} + \widehat{P_{t+1}} + \widehat{Q}_{t+1})\} \end{aligned} \quad (35)$$

where $B_1 = \frac{-\varepsilon\phi}{1-\gamma}$. Here $\widehat{w}_t, \widehat{H_t}, \widehat{P}_t$,and $\widehat{Y}_t$, represent percent deviations from steady state for $w(s^t), H(s^{t-1}), P(s^t)$ and $Y(s^t)$ respectively and E refers to the expectation at $t-1$. Note that learning-by-doing generates a pricing equation that involves one period ahead leads in marginal costs as did the *staggered pricing model*. Note also the role of organizational capital in reducing marginal costs. This expression can be further simplified by substituting out $\widehat{Q}_{t+1} = \sigma(\widehat{Y}_t - \widehat{Y_{t+1}}) + \widehat{P}_t - \widehat{P_{t+1}}$ which leaves us with:

$$\begin{aligned} E\{B_1(1-\sigma)\widehat{Y}_t - E(\widehat{w}_t - \epsilon\widehat{H_t})\} &= E\{-(\epsilon\phi+\gamma)E(\widehat{w_{t+1}} - \epsilon\widehat{H_{t+1}}) + \\ &\quad B_1(1-\sigma)(\widehat{Y_{t+1}})\} \end{aligned} \tag{36}$$

The equivalent equation in the *staggered pricing model* is

$$\widehat{P}_t = (\frac{1}{2})E(\widehat{P_{t-1}} + \widehat{P_{t+1}}) + (\widehat{w}_t + \widehat{w_{t+1}}). \tag{37}$$

Following the same steps as before we obtain an expression for $\widehat{w}_t$ in the lbd model:

$$\widehat{w}_t = \Lambda\widehat{Y}_t - \epsilon\chi\widehat{H_t}. \tag{38}$$

Recall that the presence of organizational capital (H) in the production function means that an additional state variable remains in the model even if capital is removed. This state variable captures the accumulated effect of all past levels of output as can be seen from the linearized accumulation equation for H

$$\widehat{H_t} = \gamma\widehat{H_{t-1}} + \phi\widehat{Y_{t-1}} = \frac{\phi}{1-\gamma L}\widehat{Y_{t-1}}. \tag{39}$$

where L is the lag operator. As a result we can express the relationship between wages and output as

$$\widehat{w}_t = \Lambda\widehat{Y}_t - \epsilon\chi\phi[\widehat{Y_{t-1}} + \gamma\widehat{Y_{t-2}} + \gamma^2\widehat{Y_{t-3}} + ...]. \tag{40}$$

Note that the contemporaneous relationship between output and wages (or costs) remains unchanged in the presence of learning-by-doing. The difference arises from the presence of additional lags of output. These lagged terms are present because output increases in the past lower current costs for firms and the persistence of these effects is governed mainly by γ. The higher γ is, the bigger the influence of $\widehat{Y_{t-2}}$ (for example) on current costs. On the other hand, a bigger value of γ implies that the infinite series in the square brackets decays at a faster rate. The influence of past output on costs is also determined by the other parameters in the learning technology, ϵ and ϕ. Increases in both raise the magnitude of the dynamic interlinkage without altering the rate of decay.[13] Similarly increases in steady state hours, N, and σ, the parameter governing curvature of preferences, both increase the magnitude of the dynamic interlinkage. Table 1 presents calculated values for the coefficients on some of the Y_{t-i} terms in the wage equation (40) based on some reasonable values for the underlying parameters. In all the rows of table 1 $\sigma = 1$ and $N = 1/3$ therefore $\chi = (N/1-N)\varrho = .5$. since $\varrho = \sigma$ in our calibration. As a result $\Lambda = \sigma + \chi = 1.5$. Since we have fixed $\phi = 1$ in our calibration, the coefficients associated with the lagged values of Y depend only on χ, ϵ and γ as given in the wage equation above. The first three rows of Table 1 calculates these for three different combinations of the latter two parameters. The fourth row offers the equivalent values for the staggered pricing model without lbd discussed above.

Returning to our derivation, substituting in for wages in our pricing equation yields:

$$E\{(B_1(1-\sigma)-\Lambda)\widehat{Y}_t + \epsilon(1+\chi)\widehat{H}_t)\} = E\{(\epsilon\phi+\gamma)\epsilon(1+\chi)\widehat{H_{t+1}} + (B_1(1-\sigma)-(\epsilon\phi+\gamma)\Lambda)\widehat{Y_{t+1}}\}. \quad (41)$$

Note the LBD parameters appear in the pricing equation both through B_1 and otherwise even before we account for their influence on wages. As discussed earlier, this occurs because firms wish to optimally control their learning through prices.

Replacing $\widehat{H_t}$ and $\widehat{H_{t+1}}$ using (39) and multiplying through by $1-\gamma L$, and applying the lag operator as appropriate we get our expression for output

$$d_1 E\widehat{Y_{t+1}} + d_2 E\widehat{Y}_t + d_3\widehat{Y_{t-1}} = 0 \quad (42)$$

and finally replacing $\widehat{Y}_t$ with $\widehat{M}_t - \widehat{P}_t$ we get the expression for prices:

[13] Given the symmetric effect of the two parameters, I only explore the impact of ϵ,holding ϕ fixed. This is also consistent with my calibration in the next section.

$$d_1 E\widehat{P_{t+1}} + d_2 E\widehat{P}_t + d_3\widehat{P_{t-1}} = d_1 E\widehat{M_{t+1}} + d_2 E\widehat{M}_t + d_3\widehat{M_{t-1}} \tag{43}$$

where $d_2 = B_1(1-\sigma) - \Lambda + \gamma[B_1(1-\sigma) - (\epsilon\phi+\gamma)\Lambda] - \phi((\epsilon\phi+\gamma)\epsilon(1+\chi))$, $d_1 = -B_1(1-\sigma) + (\epsilon\phi+\gamma)\Lambda$ and $d_3 = (1+\chi)\phi\epsilon - \gamma(B_1(1-\sigma) - \Lambda)$.

Comparing this expression to the equivalent expression for the staggered pricing model reveals that the two equations have a similar second order difference equation structure. Apart from key differences in coefficients, which I discuss below, note also that learning-by-doing involves not only a lead in the exogenous money process but also a lag which is missing there. The equivalent expression in the *preset pricing model* is $(\sigma+\chi)(\widehat{M}_t - \widehat{P}_t) = 0$ which implies prices would follow a random walk if $\widehat{M}_t$ did.

The difference in the coefficients in (30) and (43) play a crucial role in changing the dynamics of prices in the lbd model relative to the staggered pricing model. Chari et al (2000) show that the dynamics of their system is controlled by Λ, the only parameter appearing in (30), which in turn only depends upon preference parameters. The coefficients in (43), however, also depend on the parameters in the learning technology in complicated ways. The dynamics of the system depends on κ, the stable root of the characteristic quadratic associated with (43) where $\kappa = -d_2 \pm \sqrt{d_2^2 - 4d_1d_3}/2d_1$.

Glancing back at (42) reveals a second order homogenous difference equation in output. The solution to this system may be written as $\widehat{Y}_t = \kappa\widehat{Y_{t-1}}$ which implies that the internal dynamics of the system and therefore the persistence properties are governed by κ. I have calculated the value of κ for $\sigma = \phi = 1, N = 1/3$ as in Table 1 and allowed $\gamma \subseteq [.1, .8]$ and $\epsilon \subseteq [.1, .3]$ which encompasses the empirically relevant range discussed in the next section. For all these values κ is positive taking values between .3 and 1. For example when $\gamma = .5$ and $\epsilon = .2$, $\kappa = .7$.The contrast with the stable root implied by Chari et al for the same parameterization is stark, it has a value of $-.1010$. The results of this section suggest that learning-by-doing plays an important role in increasing the inertia of aggregate output and also of the price level. Comparing the pricing equations also suggests that one-period price-setting behaviour by firms in the lbd model mimics a two-period staggered price setting environment as in Taylor (1980) with some additional dynamics coming from the lagged money shock term. From a quantitative perspective though, the key to matching the degree of inertia seen in the data, is the value of κ. Chari et al. (2000) argue that since Λ must be greater than unity, the stable root in the staggered pricing model must be negative in their model. Despite using the same preferences (and therefore the same value for Λ) as Chari et al, the presence of additional lbd parameters allows κ to be positive in the lbd model. From the perspective of this argument, learning-by-doing succeeds in generating realistic levels of inertia because the additional learning parameters break the tight link between κ and Λ seen in Chari et al. This comes partly from the additional lagged output terms in the wage equation which links back to the idea presented in the introduction that marginal costs fall in response to a rise in money growth, and partly to the presence of organizational capital terms in the pricing equation which in turn lead to counter-cyclical markups.

3.3 The habits model

Having explained how the lbd model generates inertia in prices I now turn to deriving similar expressions for the habits model. The starting point for this exercise is the familiar linearized pricing equation for the preset pricing model

$$\widehat{P_{it}} = E(\widehat{P}_t + \widehat{w}_t). \tag{44}$$

Next we linearize the hours first order condition to get an expression for $\widehat{w_t}$ after making the usual substitutions:

$$\widehat{w}_t = \sigma \widehat{Y}_t + \frac{\chi}{(1-b)^2}[\widehat{Y}_t(1+b^2) - b\widehat{Y_{t-1}} - b\widehat{Y_{t+1}}]. \tag{45}$$

Like LBD, the presence of habits also modifies the relationship between wages and output in a dynamic way. Unlike LBD we now see a lead of output and only one lag instead of an infinite series. Moreover the contemporaneous relationship is also changed due to the presence of b. The last two rows of table 1 reports values for these coefficients taken from (45). All preference parameters other than $b(= .5$ and$.8)$ are the same as before therefore $\chi = .5$ and $\sigma = 1$. Higher values of b increase the magnitude of all three coefficients thus increasing the dynamic interlinkages between output and wages.

Replacing (45) in (44) and imposing equilibrium yields

$$E[e_1\widehat{Y}_t + e_2(-b\widehat{Y_{t-1}} - b\widehat{Y_{t+1}}) = 0 \tag{46}$$

and then replacing $\widehat{Y}_t$ with $\widehat{M}_t - \widehat{P}_t$ gives us

$$e_1\widehat{P}_t = E\{e_1\widehat{M}_t + e_2[b\widehat{P_{t-1}} + b\widehat{P_{t+1}} - b(\widehat{M_{t+1}} - b\widehat{M_{t-1}}]\} \tag{47}$$

where $e_1 = \sigma + e_2(1 + b^2)$ and $\mathrm{e}_2 = \frac{\chi}{(1-b)^2}$.

Once again we have a second order difference equation in output (46) and prices (47) which has a structure similar to the lbd model. In this case the dynamics appear from the wage equation only whereas in the lbd model, both the wage equation and the pricing equation were dynamic (due to the presence of organizational capital terms in the former). The stable root of the associated quadratic is now a function of the $\mathrm{e}_i's$ which in turn depend upon σ, N, and b. Holding the first two at the same values as in previous models, we can calculate how the stable root κ_h changes with b. It is strictly increasing in b, taking a value of .3139 when $b = .5$ and .6805 when $b = .8$. As in the lbd model, the persistence of output increases with the magnitude of κ_h.[14]

While, the results of this section are instructive, they are based on unrealistic restrictions which I drop in the following sections.

4 Computation method and calibration

The full model with physical capital is solved using the method outlined in King and Watson (2002) using a linear approximation to the system of equations including the first order conditions of the intermediate goods producers problem, the first order conditions from the consumers problem, the production function, the resource constraint for the economy and the accumulation equation for physical and organizational capital. Some variables are growing in steady state - they are rendered stationary by dividing by the stock of money in the economy.

In order to simulate the economy, functional forms have to be specified. With the exception of the presence of habit formation in leisure, the specification for preferences is similar to Chari et al (2000):

$$U(C, L, M/P) = \frac{1}{1-\sigma}[(\omega C^{(\theta-1)/\theta} + (1-\omega)(\frac{M}{P})^{(\theta-1)/\theta})^{\frac{\theta}{\theta-1}} L^{*\psi}]^{1-\sigma}. \quad (48)$$

Here $L_t^* = L_t - bL_{t-1}$ whenever we allow for habit formation in leisure but simplifies to L_t in the benchmark case where the model only has learning-by-doing. The parameter b governs the extent to which habits are formed. A

[14] I have not reported the results for the case when both lbd and habits are present in the model. The presence of both these mechanisms gives rise to a fourth order difference equation in prices.

number of estimates of b are available in the literature ranging from a high of roughly 0.8 in Eichenbaum, Hansen and Singleton (1988) to a low around 0.5 in Braun and Evans (1998). For the calibration of the model with both lbd and habits which I will refer to as the *full model*, I picked the midpoint value of $b = .65$. Sensitivity analysis with $b = .5$ is also provided. Other parameters related to preferences are taken from Chari et al. I set $\omega = .94$, and $\theta = .39$ and set ψ so that the fraction of the time endowment spent on working in steady state is .3.

The typical value for σ, in the literature is unity. However, non-separability in leisure contributes to generating inflation inertia. In order to stay close to unity and yet allow for non-separability I set $\sigma = 1.1$. This value of sigma was used for all models with habits in leisure including the specification referred to in the next section as the full model. The auto-correlation coefficient of inflation, ρ_π, in the full model (but not output) is sensitive to small changes in σ. As σ goes from 1 to 1.1, ρ_π goes from about .6 to .8. This sensitivity to σ is much less acute in the absence of habits. Nonetheless, higher values of σ deliver bigger ρ_π as well as less investment volatility. If σ is too high it is impossible to generate enough volatility in investment even without adjustment costs. Since the other models are calibrated to the relative volatility of investment seen in the data, it was important to restrict σ so that a similar calibration could be done across all models. Therefore for the benchmark model without learning or habits, σ is chosen so that the model is able to deliver the correct relative volatility of investment . This value ($\sigma = 4$) is kept constant when comparing the benchmark and lbd models *without habits*. Sensitivity to σ for the lbd model is also provided. In all cases, the investment adjustment cost parameter, v, is set to keep the ratio of the standard deviation of investment and output equal to their value in the US data.

Following Cooper and Johri (2002), β, the discount factor is set to .984 while δ, the depreciation rate is set to .02, the value estimated in Johri and Letendre (2007). The parameter η is chosen in all models to maintain a steady state markup of 5%. The results are not very sensitive to changes in the steady state markup.

Turning to the specification of technology, intermediate goods producers are assumed to use a Cobb Douglas production function to produce output, given by

$$Y_i(s^t) = N_i^{\alpha}(s^t) K_i^{1-\alpha}(s^t) H_i^{\varepsilon}(s^{t-1}). \tag{49}$$

while the accumulation equation for organizational capital is

$$H_i(s^t) = H_i^{\gamma}(s^{t-1})Y_i^{\phi}(s^t). \tag{50}$$

The elasticity of output w.r.t. physical capital is set to .39 to deliver a capital output ratio of 10.24 which is the value seen in the data and α is chosen to maintain constant returns in the benchmark model without learning-by-doing.

Turning to the parameters associated with learning-by-doing, I provide results for two set of values for the elasticity of output with respect to organizational capital, ε. These values corresponds to a "learning rate" of either fifteen percent or ten percent. Note that these learning rates are much lower than commonly estimated in microeconomic studies in the traditional learning-by-doing literature. The "consensus" estimate based on an extensive list of industries over the past hundred or so years appears to be twenty percent learning. See Irwin and Klenow (1994) for estimates in the semi-conductor industry as well as a discussion of past studies. Note that these studies impose quite strong restrictions on the accumulation of organizational capital which is governed by

$$H(t+1) = H(t) + Y(t).$$

In particular, note that each unit of past output contributes equally (a value of unity) to the accumulation of H(t+1) through H(t) no matter how long ago it was produced. To the extent that knowledge gleaned from past production is either lost to the firm because of labour turnover or re-organizations or becomes increasingly irrelevant over time, imposing a value of unity on the contribution of H(t) seems overly restrictive. Both Benkard (2000) and Cooper and Johri (2002) drop this restriction. Benkard shows that allowing for "organizational forgetting" leads to higher estimates of the learning rate. For example in his work on aircraft production, the estimated learning rate rises from roughly 20 percent to 39 percent once "organizational forgetting" is allowed. Keeping this in mind, and the even higher estimate of $\varepsilon = .49$ for the aggregate economy in Cooper-Johri (2002), suggests that the amount of learning-by-doing built into the model is fairly conservative.

Estimates of γ range between .5 and .8. I set $\gamma = .5$ and explore the impact of raising it to $\gamma = .75$. The parameter ϕ is typically set to unity in the literature which is also the case here. As discussed in the previous section, ϕ and ϵ play similar roles in propagating shocks. In the next section the plots labeled lbd model are based on $\gamma = .5$ and a learning rate of fifteen percent.

5 Dynamics in the full model

The main question addressed in this section is how much additional inertia in inflation and aggregate output can be generated by adding learning-by-doing and habits in leisure to the one period sticky price model. In addition, the responses of the model economies are compared to aggregate US data when the artificial economies are hit by estimated money growth and tfp shocks identified from US data. Theoretical impulse responses to money growth shocks can be found in the appendix.

The data for consumption, output, hours worked and investment are all seasonally adjusted US quarterly time series, from 1960:II to 2008:III. Specifically I use real consumption of non-durables and services in chained 2000 dollars, total hours for non-agricultural industries, real gross private domestic investment in chained 2000 dollars and real gross domestic product in chained 2000 dollars. All series are expressed in per capita terms by dividing by the civilian non-institutional population, ages 16 and over. Inflation and growth rate of money are measured by annualized, quarter-to-quarter percentage changes in the GDP implicit price deflator and continuously compounded rate of change of M2 money stock from the St. Louis' Fed website, respectively. The innovations of growth rate of money is measured by fitting an AR(1) process to the growth rate of M2 after removing a linear trend. The fitted equation is $\mu_t = \rho_m \mu_{t-1} + \upsilon_{mt}$. The tfp shocks are calculated by first estimating a standard rbc model using bayesian techniques. The estimated shock process is fitted to an AR(1) regression and innovations are fed into the model.

5.1 Business cycle moments

Table 2 reports unfiltered theoretical second moments for various economies. The first row reports results for the one-period sticky price model with neither lbd nor habits. Rows 2-5 report results for the lbd model as various key parameters are changed. This sensitivity analysis is followed by the full model in row 6. Rows 7 and 8 focus on the habit mechanism only. Row 9 provides the relevant log-linearly detrended moments calculated from aggregate US data for comparison.

Relative standard deviations of consumption and hours with output are reported in columns 1 and 2 of Table 1. These are followed by contemporaneous correlations between output and average labour productivity, consumption, investment and hours, in that order. The last two columns report the first order auto-correlation coefficient on output and inflation, denoted ρ_y and ρ_π respectively. In all cases, the adjustment cost parameter has been set so that the standard deviation of investment to that of output matches the equivalent empirical moment.

Looking through the rows of Table 1, all the models do a good job of capturing the basic features of business cycles. Consumption, hours and investment are all procyclical and there is evidence of consumption smoothing. The models also inherit some common problems: in all cases consumption is more volatile than in the US data. In fact the behaviour of consumption is virtually identical in the models. Similarly, investment is somewhat more correlated with output in all models relative to the data. Clear differences, however, appear across models when we study the behaviour of hours. The models without learning-by-doing tend to generate too much volatility in hours relative to output as compared to the data. Interestingly habit formation in leisure does not help lower the relative volatility of hours very much. Learning-by-doing also plays an important role in reducing the negative comovement between average labour productivity and hours. As the learning rate rises from zero in the first row to fifteen percent in row five, this correlation rises from -.50 to -.27. However it still falls short of .08, the value seen in the data. Increasing the persistence of habits appears to worsen the behaviour of hours both in terms of its correlation with productivity as well as it's volatility relative to output.

I now turn to a discussion of the correlation between inflation and its first lag (ρ_π) as well as output and its first lag (ρ_y). In the US data, both inflation and output are highly auto-correlated ($\rho_\pi = .88$ and $\rho_y = .97$). The benchmark sticky price model in row one is unable to generate this much persistence in either variable. The simulated values for these moments are $\rho_\pi = .07$ and $\rho_y = .73$. As expected, the inclusion of learning-by-doing in the model leads to significant increases in persistence ranging from .27 to .76 for ρ_π. Similarly ρ_y ranges between .84 and .87 as various parameters are varied. Inflation persistence is increasing in σ and ϵ while it is decreasing in γ, while output persistence is increasing in both γ and ϵ and decreasing in σ but is not very sensitive to any of these parameters. I conclude from these sensitivity exercises that learning-by-doing is an effective mechanism at generating persistence in both variables but for the conservative levels of learning explored here, is unable to fully account for either.

Row 6 reports results for the full model with moderate learning effects as well as moderate habit formation. The model can account for all the persistence in inflation and most of the persistence in output($\rho_\pi = .88$ and $\rho_y = .94$). The next two rows attempt to study the contribution of the habits channel on its own. Looking down the two rows we see that increasing the habit formation parameter b has a big impact on inflation and none on output. Moreover, on its own, the habits model is able to account for roughly the same level of persistence as the lbd model in both variables.

Additional intuition for the mechanisms on display can be found by looking at the model generated impulse response of key variables (not shown) to a one percent increase in the growth rate of money. These responses can be found in the appendix. The full model generates considerably more inflation inertia than the benchmark model with neither lbd nor habits for two reasons. First, due to the presence of learning-by-doing and habit formation, marginal costs rise very slowly. Second, firms may wish to take advantage of the high demand for their product to learn, and therefore raise prices by less than the increase in marginal cost. In other words, firms lower their markups. The impulse responses show that the behaviour of costs in the two models is quite different. In the baseline model nominal costs increase faster than money supply while in the full model nominal costs increase slower than the money supply. Moreover the additional influence on price inertia exerted by time-varying markups is also visible in the Full model plots. Firms lower their markup below steady state levels in the periods immediately after the shock occurs. As organizational capital is accumulated, it's marginal value falls so that after three periods firms find it more profitable to raise markups slightly.

5.2 Comparing model based time series to the data

While the average response of output and inflation generated by the model economies seems to be quite good, in this section I will compare the entire time-series generated by three models with the corresponding time-series from the US. The first model is called the benchmark model. This model has neither lbd nor habits. The second model is the calibrated lbd model without habits. Once habits are included, we get the third model which is referred to as the Full model. The benchmark model corresponds to row one of table 2. The lbd model corresponds to row five and the full model to row six of the same table. This exercise is similar to that of McGrattan (2004).[15] The plots for inflation and output are reported in Figures 1 and 2. Figure 1 shows how the models respond to tfp shocks alone and Figure 2 shows how they respond to both shocks together. The time-series for consumption, hours and investment can be found in an appendix available on the journal web-site.

Figure 1 plots the response of inflation and output in various models to a tfp only shock. Figure 1a and 1b plot the response of the benchmark sticky price model and the learning-by-doing model with a 15 percent learning rate against inflation and output data. Clearly the LBD model reduces the volatility of inflation and increases that of output. Figures 1c and 1d show that the full model further helps to lower the volatility of inflation and output. To get a complete sense of the performance of the models we plot their responses to both tfp and M2 growth shocks in figure 2. The ability of the lbd model to temper the volatility of inflation is clearly visible in figure 2a. This is in line with the data in which the relative volatility of inflation to output is .18. Absent learning by doing, the benchmark model has a relative volatility of .47, which falls to .17 in the LBD model both because the volatility of inflation falls and that of output rises as seen in figure 2b. These moments are reported in the first column of Table 3. Another way to assess how well the models explain the data is to calculate the correlations of the actual time-series with the artificial data. As one might expect from Figure 2, both models explain output better than inflation. The correlations for the benchmark model without learning-by-doing are .79 and .27 while they are .73 and .43 for the LBD model.[16]

Figure 2c and 2d show the impact of adding habits to the LBD model which we refer to as the full model. This leads to a somewhat lower relative volatility number for the full model compared to the lbd model (.14 vs. .17). As captured by the moments previously discussed, the addition of habits not only lowers the volatility of inflation and output, it also increases their persistence. These changes in the dynamics of the model result in only minor variation in its ability to account for variations in the data. The last two

[15]I thank Ellen McGratten for encouraging me to do this exercise.

[16]Sensitivity analysis of the lbd model plots to parameter changes are available from the author and have a very minor impact. Plots for variations of the habits model discussed in table 2 are also available.

rows of table 3 show that the habit channel on its own does no better at capturing movement in the data.

6 Conclusions

Learning-by-doing and habit formation in leisure are introduced into a monetary dynamic general equilibrium model. In order to highlight the ability of the model to generate inertia in the aggregate price level, all other sources of inertia commonly used in the literature such as menu costs, staggered price or wage contracts are ignored. The model therefore relies on the minimal amount of price stickiness needed: prices are chosen before the shocks occur. A calibrated version of the model generates considerable inertia in both inflation and output dynamics in response to money growth and tfp shocks. The model also does reasonably well in matching moments that capture key features of the US business cycle.

Despite this success, many problems remain. In particular, time series of simulated inflation based on estimated shocks to money growth and tfp fail to closely match the actual behaviour of inflation. The sticky price model without either learning-by-doing or habit formation tends to predict an inflation series that is much more volatile than the data and much less persistent. Both of the mechanisms explored in this paper help in this respect: increasing persistence and lowering volatility but predicted series remain much more volatile than the actual data. Comparatively speaking, the models do much better in replicating the behaviour of aggregate output than inflation. This could, in part, be due to the modelling of monetary policy as an exogenous autoregressive process. Moreover many other features that have been shown to help bridge the gap between model and data in the literature are missing here. Exploring the interaction of these missing features with learning-by-doing and habit formation may be a fruitful avenue for future research.

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Table1-3

	Y_{t+1}	Y_t	Y_{t-1}	Y_{t-2}	Y_{t-3}	Y_{t-4}
LBD: $\epsilon = .2, \gamma = .5$	0	1.5	-.1	-.05	-.025	-.0125
LBD: $\epsilon = .2, \gamma = .8$	0	1.5	-.1	-.08	-.064	-.0512
LBD: $\epsilon = .1, \gamma = .8$	0	1.5	-.05	-.04	-.032	-.0256
2 quarter staggered	0	1.5	0	0	0	0
Habits(b=.5)	-1	3.5	-1	0	0	0
Habits(b=.8)	-10	21.5	-10	0	0	0

Table 1: Wage equation coefficients

	$\sigma_{C/Y}$	$\sigma_{N/Y}$	$\rho_{alp/N}$	$\rho_{C/Y}$	$\rho_{I/Y}$	$\rho_{N/Y}$	ρ_Y	ρ_π
no lbd, no habits[1]	0.94	.97	-.46	0.99	0.93	0.52	0.73	.07
10% LR,$\sigma = 2$	0.91	.68	-.24	0.99	0.99	0.46	0.87	.27
10% LR,$\sigma = 4$	0.92	0.76	-.27	0.99	0.97	0.52	.84	.52
10% LR,$\sigma = 4, \gamma = .75$	0.99	0.76	-.29	.99	0.97	0.48	0.84	.36
15% LR,$\sigma = 4$	0.92	0.76	-.20	0.99	0.97	0.55	0.87	.76
Full: 15% LR, $b = .65$	0.92	.68	-.20	0.99	0.99	0.50	0.94	.88
habits only, $b = .5$	0.91	.93	-.48	0.99	0.99	0.44	0.82	.46
habits only, $b = .65$	0.91	1.08	-.56	0.99	0.99	0.50	0.83	.72
US Data	0.79	0.72	.08	0.92	0.83	0.78	0.97	.88

Table 2: Second Moments with technology and money shocks

1.Row 1 is the benchmark model. Rows 2-5 refer to variants of the LBD model. Row 5 is the version calibrated to the US economy. Row 6 is the Full model. Rows 7 and 8 are the habits only model.

	$\sigma_{\pi/Y}$	$\rho_{Ysim/Y}$	$\rho_{\pi sim/\pi}$
no lbd, no habits	0.47	0.79	.27
10% LR	0.24	0.78	.38
15% LR	0.17	0.73	.43
Full model:15% LR, $b = .65$	0.14	0.63	.44
no lbd, $b = .5$	0.30	0.74	.30
no lbd, $b = .65$	0.23	0.66	.35
US Data	0.18		

Table 3: Comparing actual and simulated inflation and output data.

Figure1a

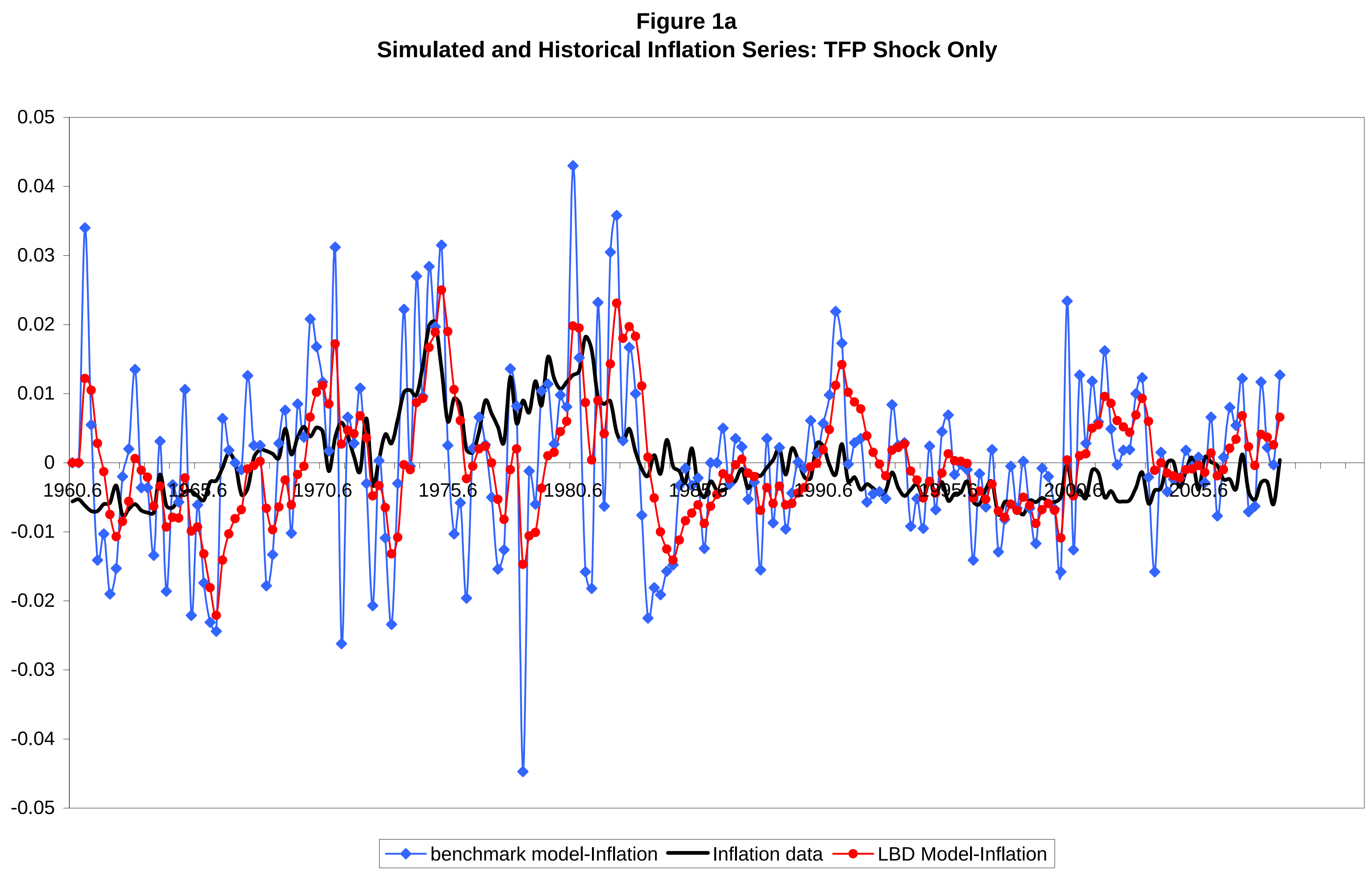
Figure 1a
Simulated and Historical Inflation Series: TFP Shock Only
0.05
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
1960.6
1965.6
1970.6
1975.6
1980.6
1985.6
1990.6
1995.6
2000.6
2005.6
benchmark model-Inflation
Inflation data
LBD Model-Inflation

Figure1b

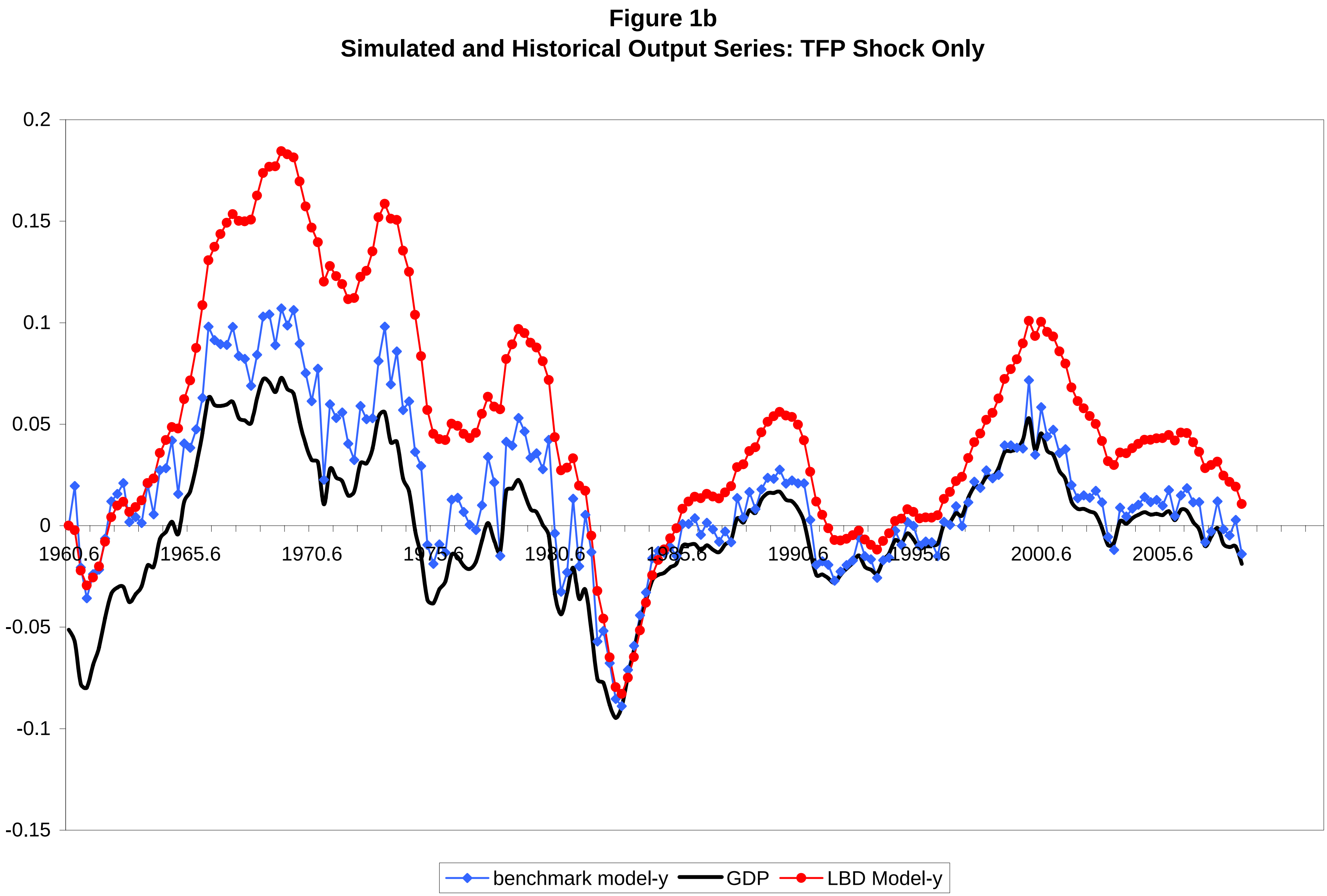

Figure 1b
Simulated and Historical Output Series: TFP Shock Only
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1960.6
1965.6
1970.6
1975.6
1980.6
1985.6
1990.6
1995.6
2000.6
2005.6
benchmark model-y
GDP
LBD Model-y

Figure1c

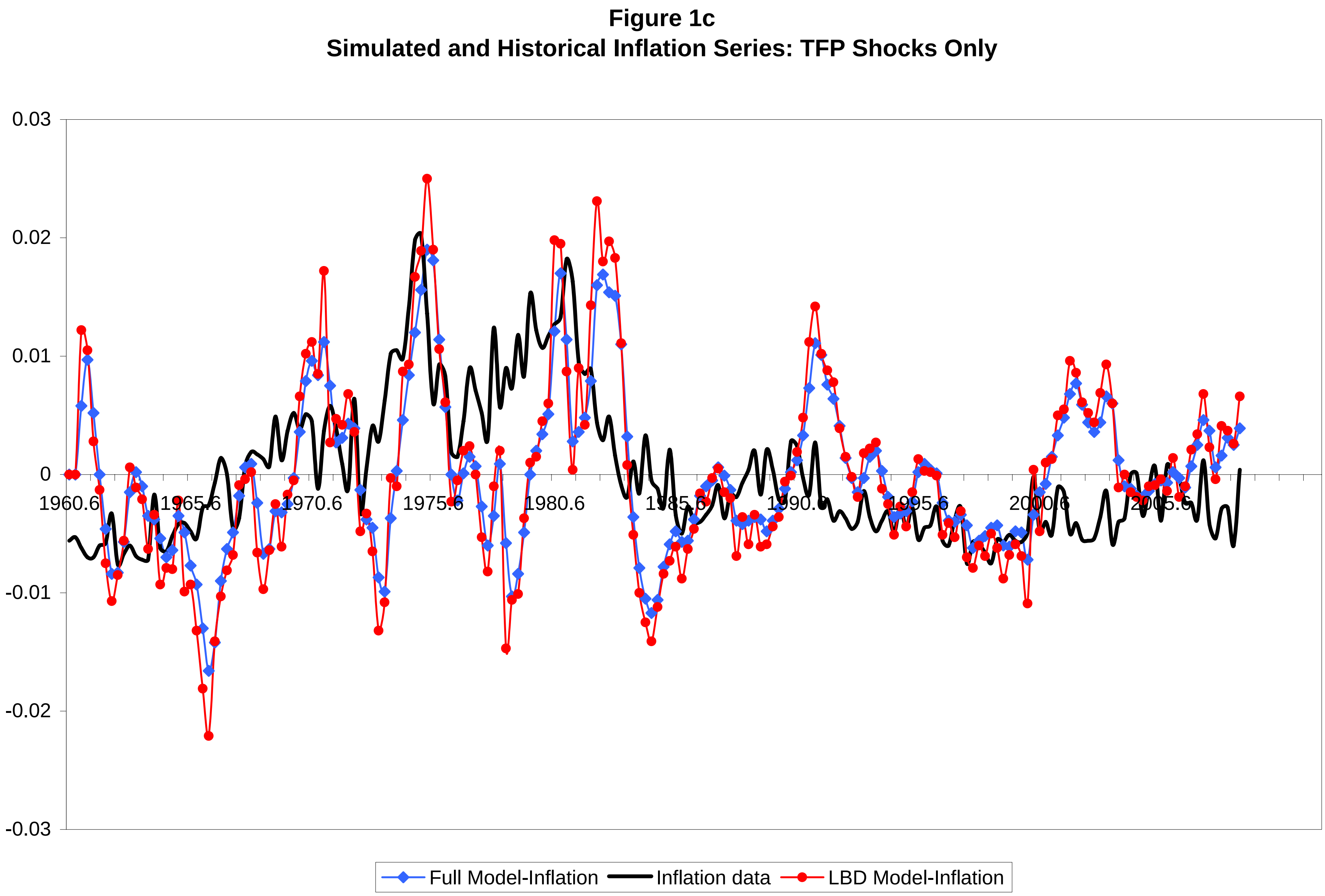
Figure 1c
Simulated and Historical Inflation Series: TFP Shocks Only
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1960.6
1965.6
1970.6
1975.6
1980.6
1985.6
1990.6
1995.6
2000.6
2005.6
Full Model-Inflation
Inflation data
LBD Model-Inflation

Figure1d

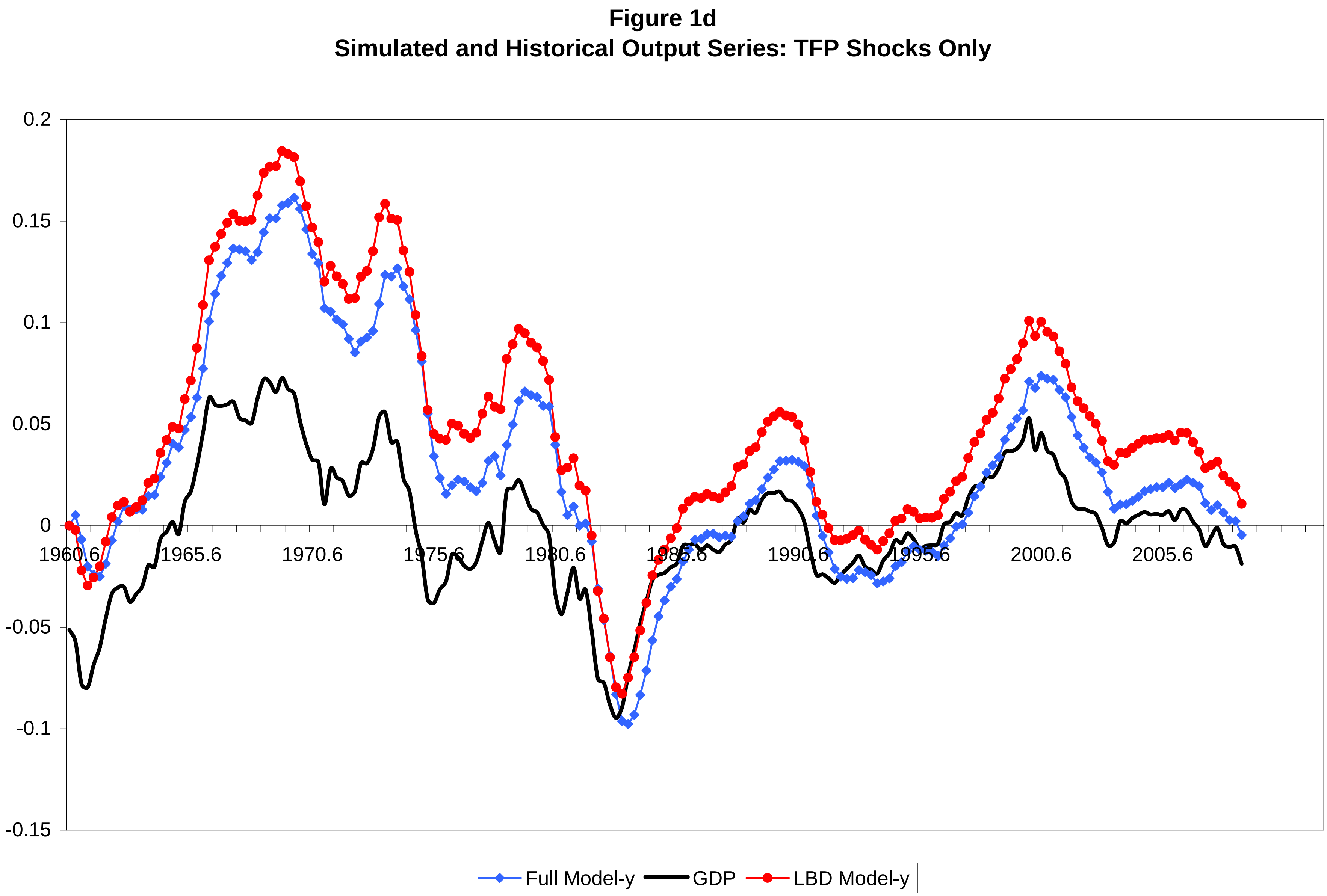
Figure 1d
Simulated and Historical Output Series: TFP Shocks Only
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1960.6
1965.6
1970.6
1975.6
1980.6
1985.6
1990.6
1995.6
2000.6
2005.6
Full Model-y
GDP
LBD Model-y

Figure2a

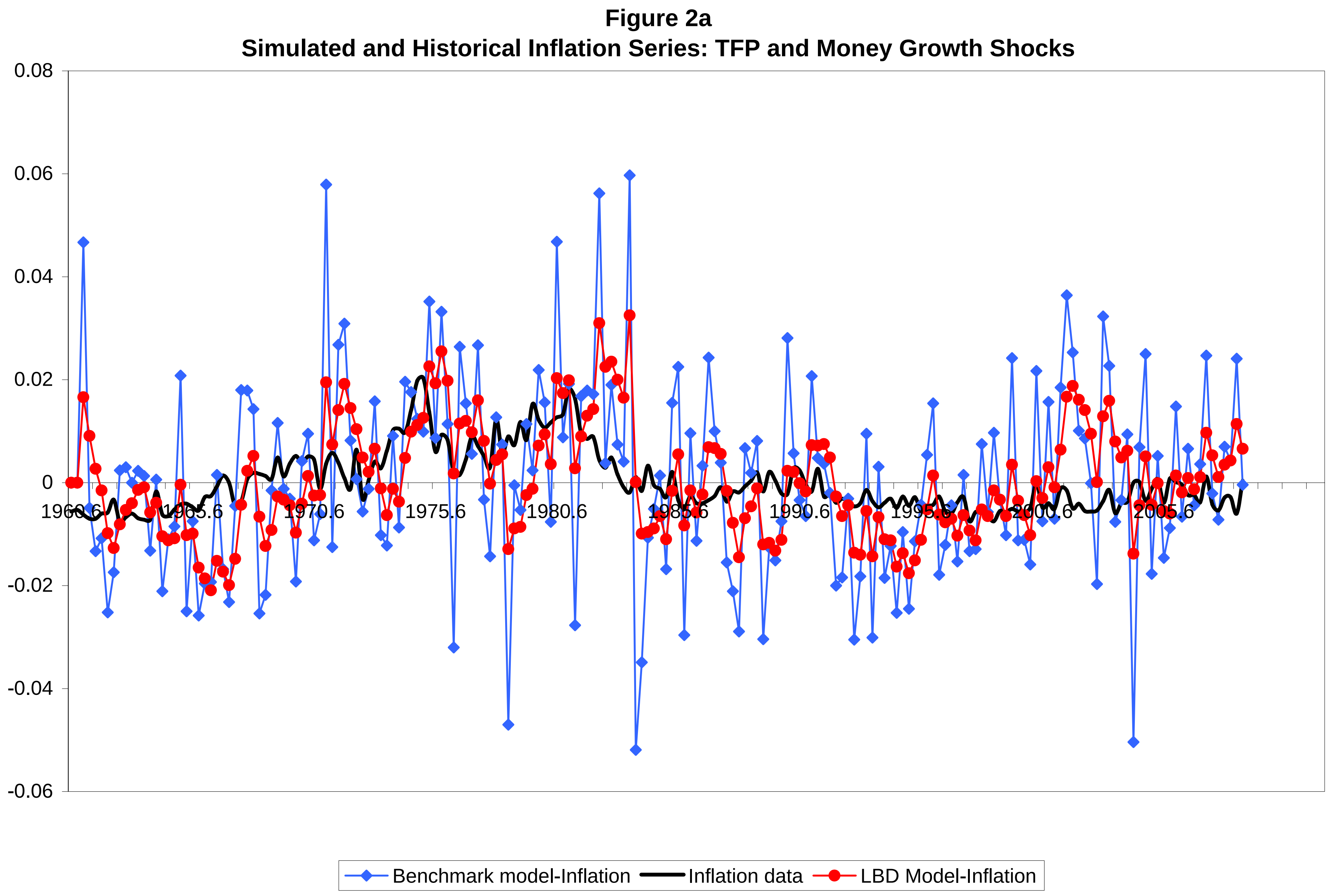
Figure 2a
Simulated and Historical Inflation Series: TFP and Money Growth Shocks
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
1960.6
1965.6
1970.6
1975.6
1980.6
1985.6
1990.6
1995.6
2000.6
2005.6
Benchmark model-Inflation
Inflation data
LBD Model-Inflation

Figure2b

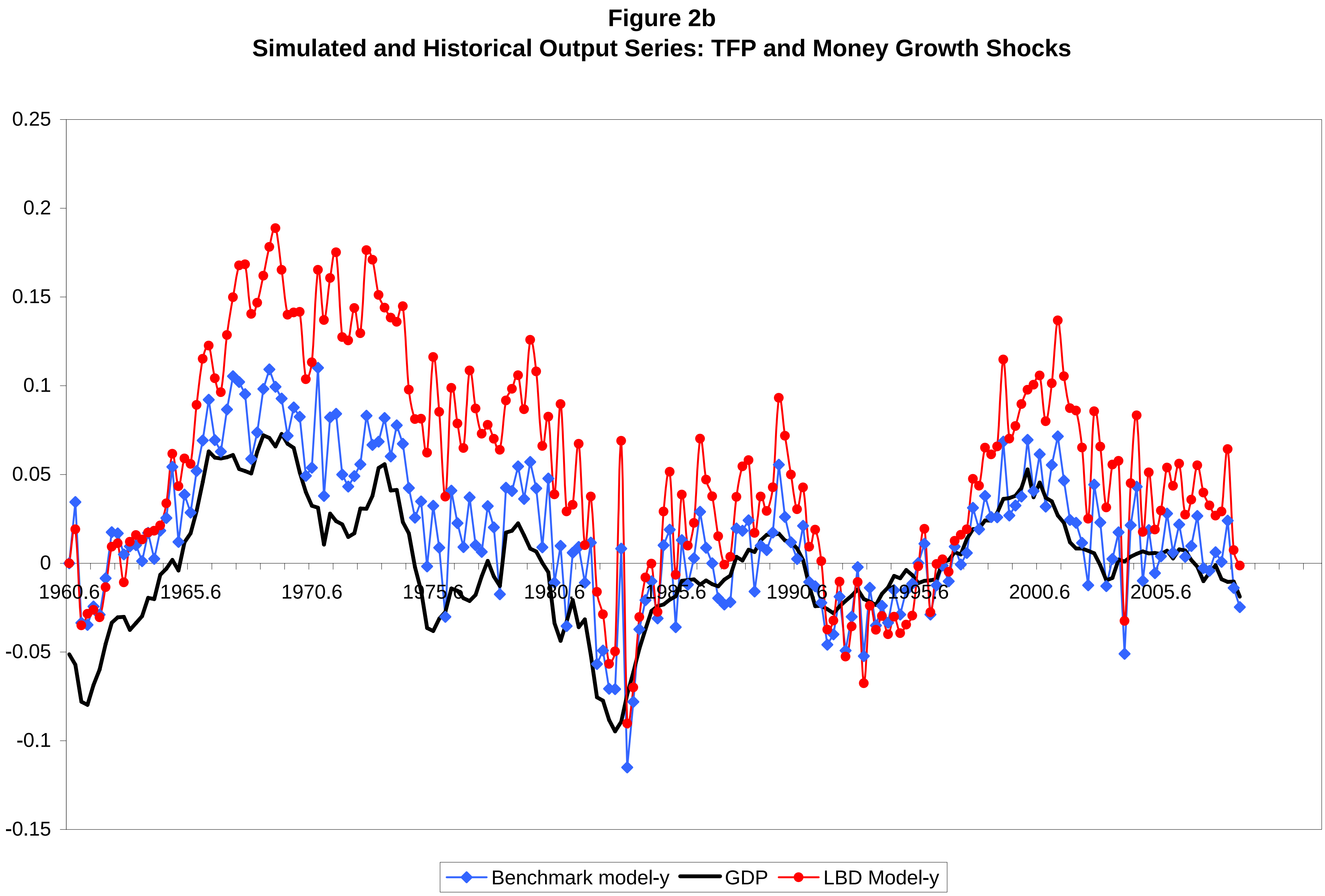

Figure 2b
Simulated and Historical Output Series: TFP and Money Growth Shocks
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1960.6
1965.6
1970.6
1975.6
1980.6
1985.6
1990.6
1995.6
2000.6
2005.6
Benchmark model-y
GDP
LBD Model-y

Figure2c

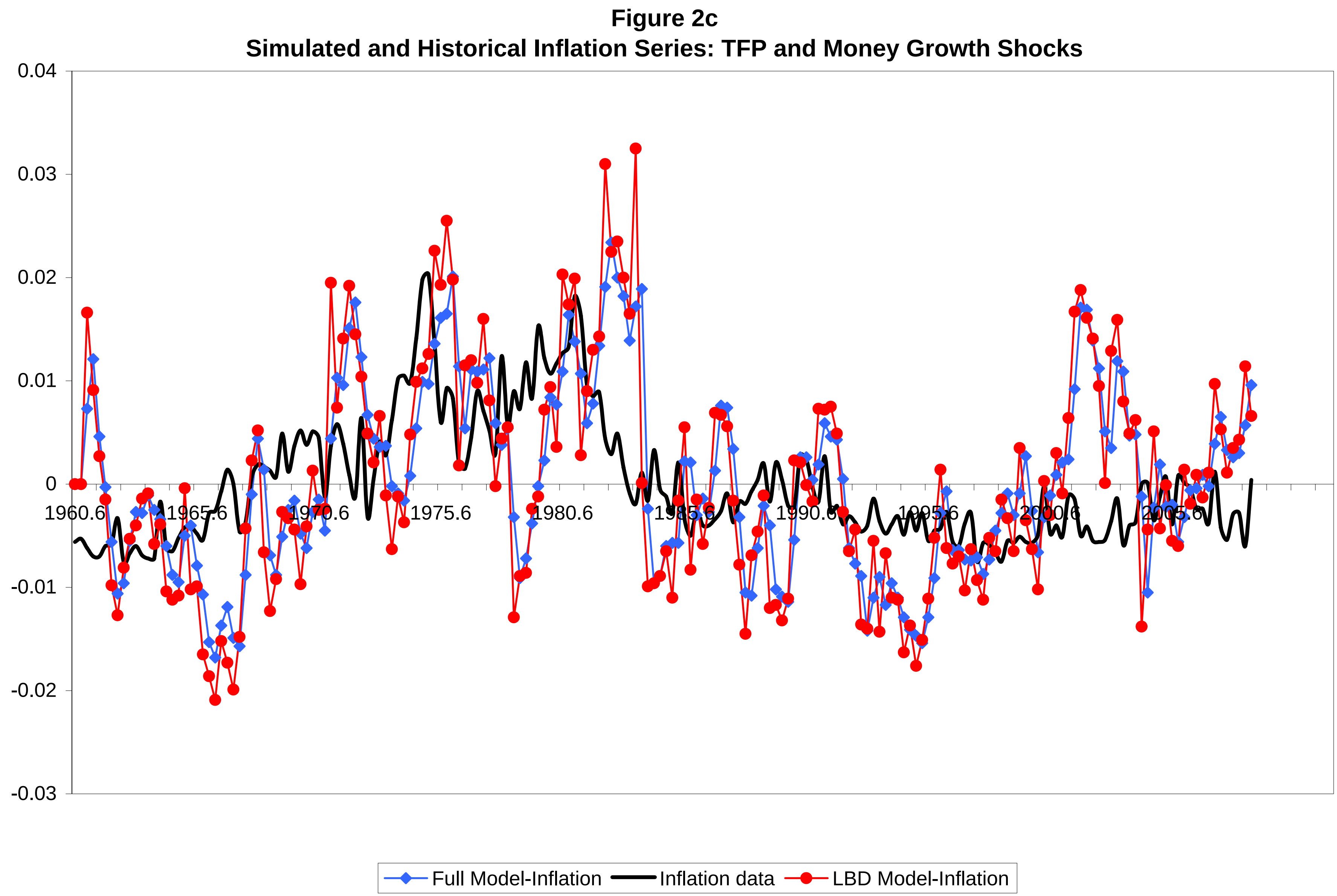

Figure 2c
Simulated and Historical Inflation Series: TFP and Money Growth Shocks
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
1960.6
1965.6
1970.6
1975.6
1980.6
1985.6
1990.6
1995.6
2000.6
2005.6
Full Model-Inflation
Inflation data
LBD Model-Inflation

Figure2d

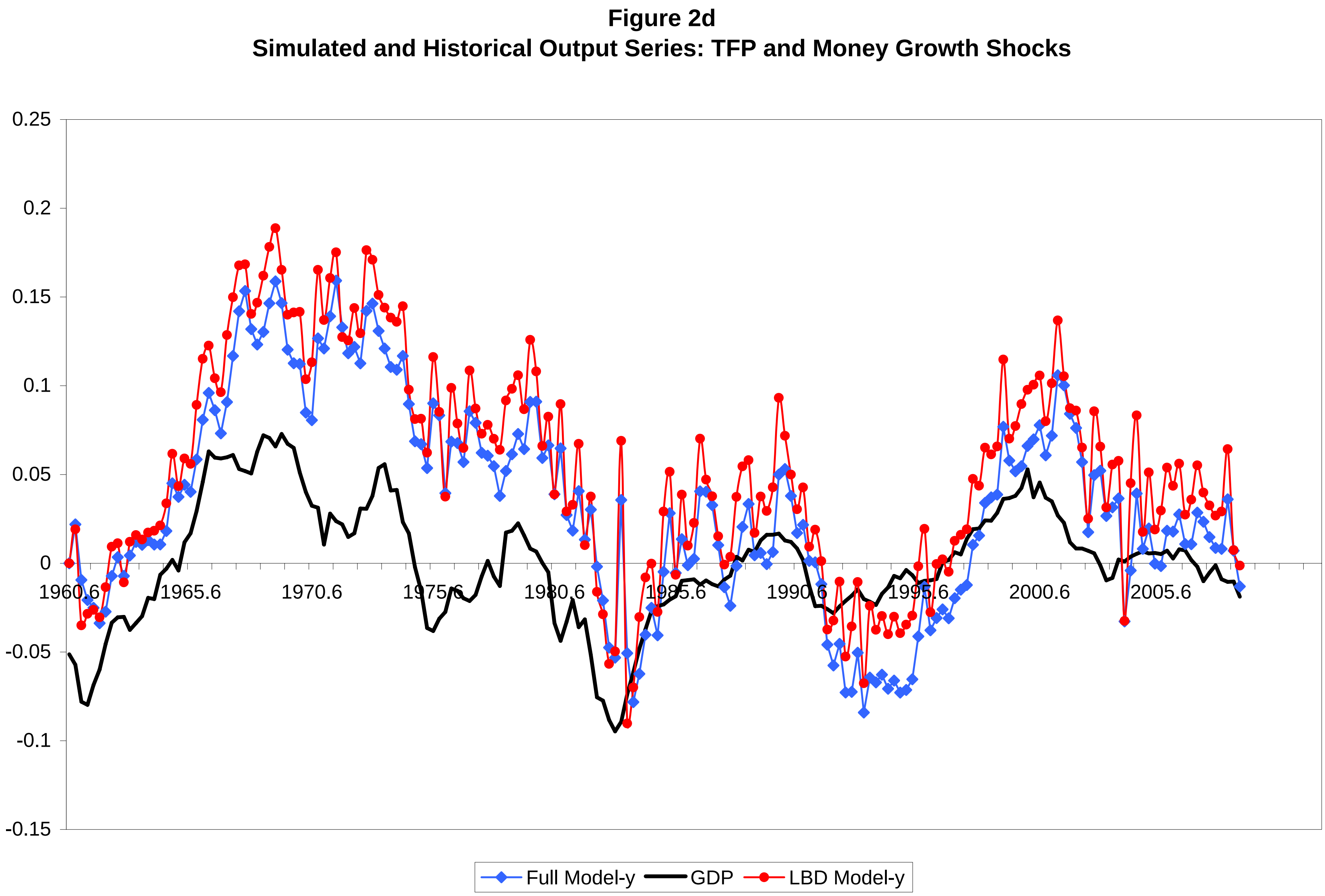
Figure 2d
Simulated and Historical Output Series: TFP and Money Growth Shocks
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
1960.6
1965.6
1970.6
1975.6
1980.6
1985.6
1990.6
1995.6
2000.6
2005.6
Full Model-y
GDP
LBD Model-y