Competition relative to Incentive Functions in Common Agency

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Abstract

In common agency problems, competing principals often incentivize a privately-informed agent's action choice by offering incentive functions that specify the principal's action as a *function* of the part of the agent's action that is contractible. This paper shows that when the agent's preference relation is strictly monotone in each principal's action, the set of all equilibrium allocations relative to incentive functions is identical to the set of all equilibrium allocations relative to any complex mechanisms.

1 Introduction

In common agency problems, competing principals try to control a privately-informed agent's action choice because their preferences on the agent's actions are not aligned with one another. Common agency problems are prevalent in practice, ranging from lobbying, public good provision, selling private goods, vertical contracting to financial contracting. The literature on common agency takes the path, as a natural route, in which each principal non-cooperatively incentivizes the agent's action choice with an *incen-*

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tive function that specifies his action as a *function* of the part of the agent's action that is contractible.[1]

The competition model relative to incentive functions however precludes the possibility of the principal offering to the agent a complex mechanism that assigns his incentive function contingent on the agent's message, or equivalently a menu of incentive functions from which the agent chooses an incentive contract that she wants, according to the menu theorem (Peters 2001; Martimort and Stole 2002). The menu theorem shows that for any equilibrium relative to any complex mechanisms, there exists an equilibrium relative to menus of incentive functions that sustains the same outcomes.[2]

The menu theorem is very general because it works without restrictions on principals' preference relations and the agent's. However, it is quite difficult to derive an equilibrium in the game where principals offer menus of incentive functions. Pavan and Calzolari (2010) showed that it is useful in deriving an equilibrium relative to menu to utilize a class of incentive-compatible extended direct mechanisms that ask the agent about her payoff type and also about her choice of payoff-relevant alternatives from the other principals.[3] The competition relative to the class of incentive-compatible extended direct mechanisms does not generate all equilibrium allocations relative to any complex mechanisms, but it does equilibrium allocations associated with Markov pure-strategy equilibria relative to any complex mechanisms in which the agent's report depends only on the payoff-relevant information.

This paper examines when it is enough to model principals' competition only relative to incentive functions in terms of characterizing the set of all equilibrium allocations relative to complex mechanisms. It shows that when the agent's preference relation is strictly monotone in each principal's action, the set of all equilibrium allocations relative to incentive functions is the same as the set of all equilibrium allocations relative to the set of any complex mechanisms that assign an incentive function contingent on the agent's message. The agent's preference relation satisfies the strict

[1] We use masculine pronouns for principals and feminine pronouns for the agent.

[2] The revelation principle for a single principal associated with incentive-compatible mechanisms defined over the agent's payoff type does not hold when multiple principals are non-cooperatively competing in the market (Epstein and Peters 1999). The reason is that the agent has private information not only on her payoff type but also on what is happening in contracting with the other competing principals.

[3] Once equilibrium incentive-compatible extended direct mechanisms are identified, each principal can equivalently offer a menu that is the image set of his equilibrium mechanism.

monotonicity in each principal's action if there exists a unique action in any compact subset of each principal's actions that maximizes the agent's payoff given the contractible part of the agent's action regardless of her payoff type, the non-contractible part of her action, and the other principals' actions. Our theorem should be useful examining when the competition relative to incentive functions generates all equilibrium allocations relative to complex mechanisms. Notably, this condition is satisfied when each principal's action can be characterized by monetary transfer between him and the agent (e.g., monetary payment, price, tax, and etc.).

Our result is closely related to Peters (2003) in that he identified the condition under which the set of *pure-strategy* equilibrium *payoffs* relative to complex mechanisms can be preserved by equilibrium payoffs relative to incentive functions. This condition is called the "no externalities" condition for principals' preference relations and the agent's. The strict monotonicity for the agent's preference relation identified in this paper satisfies the "no externalities" condition for the agent's preference relation in a stronger form but our result requires no restrictions on principals' preference relations. With the strict monotonicity condition for the agent's preference relation only, our result extends Peters's result (2003) in the sense that it allows for mixed strategy equilibria and it shows the equivalence between the set of *all* equilibrium *allocations* relative to complex mechanisms and the set of all equilibrium allocations relative to incentive functions.

It is important to know that the incentive function is not equivalent to the menu. A menu is defined as an arbitrary set of incentive functions in Peters (2001, 2003) given the viewpoint that a mechanism is a negotiating scheme that specifies an incentive function contingent on the agent's message. Clearly the set of incentive functions is a strict subset of the set of all possible menus of incentive functions because an incentive function is a menu with only one incentive function. Alternatively, one can view a mechanism as a negotiating scheme that directly specifies the principal's action and the contractible part of the agent's action contingent on the agent's message. For example, suppose that only the jth component x_j of the agent's action $x = [x_1, \ldots, x_J]$ is contractible between principal j and the agent. Given this alternative viewpoint of the mechanism, a menu offered by principal j can be defined as an arbitrary set of y_j-x_j pairs, where y_j denotes principal j's action. Any incentive function $a_j(x_j)$ can be converted into a menu by taking its graph $\{(y_j, x_j)\colon y_j = a_j(x_j)\ \forall x_j\}$. However, not every menu can be converted into an incentive function. Note that a menu can include two different pairs (y_j°, x_j) and (y_j', x_j) given the same x_j. However, such a menu cannot be converted into an incentive function because an incentive function specifies a unique y_j for any given x_j.[4] Therefore, the set of incentive functions (more precisely, the set of graphs of incentive functions) is also a strict subset of the set of all possible menus given this alternative definition of the menu.

Our paper shows that, in addition to its tractability in deriving equilibrium allocations shown in the literature, the competition relative to this particular subset of menus (i.e., the set of incentive functions) generates the set of equilibrium allocations that is the same as the set of all equilibrium allocations relative to any complex mechanisms when the agent's preference relation is strictly monotone in each principal's action. The taxation principle (Rochet 1985; Guesnerie 1995) for the case with a single principal and a single agent (or a continuum of agents) shows that the principal can rely on the set of incentive functions without loss of generality when monetary transfer (e.g, monetary payment, price, tax and etc) is involved. The result in our paper can be viewed as a *generalized taxation principle*, which provides the sufficient condition under which the set of all equilibrium allocations relative to incentive functions is the same as the set of all equilibrium allocations relative to any complex mechanisms in the environments with an arbitrary number of principals.

2 Preliminaries

When a measurable structure is necessary, the corresponding Borel σ -algebra is used. For a set S, $\Delta(S)$ denotes the set of probability distributions on S. For any $s \in \Delta(S)$, supp s denotes the support of the probability distribution s. For any mapping L from G into Q, $L(G)$ denotes the image set of L.

There are a set of principals, $\mathcal{J} \equiv \{1, \cdots, J\}$, and a single agent. The agent has private information about her preferences. This information is parameterized by an element, called a (payoff) type, in a set Ω. Principals share a common prior belief that the agent's type follows a probability distribution F on Ω. The agent can take an action x from a set X. Each principal j can take an action y_j from a set Y_j. If the agent of type ω takes an action x and the array of actions that principals take is $(y_1, \ldots, y_J)$, the agent's payoff is $u(y_1, \ldots, y_J, x, \omega) \in \mathbb{R}$ and principal j's payoff is $v_j(y_1, \ldots, y_J, x, \omega) \in \mathbb{R}$.

[4] A menu can be converted into an incentive *correspondence* that specifies a set of principal j's actions conditional on x_j. In this case, the agent chooses x_j and then principal j's action from the set specified by x_j.

An incentive function $a_j \colon X \to Y_j$ that principal j offers to the agent specifies his action as a function of the part of the agent's action that is contractible between them. Following Peters (2003), let $\mathcal{X}_j$ be a collection of measurable equivalence classes, whose union is X such that principal j is constrained to respond to each action in the same equivalence class the same way. It implies that, for any incentive function a_j, $a_j(x) = a_j(x')$ if x and x' belongs to the same equivalence class, say $\tilde{x}$: i.e., $x, x' \in \tilde{x}$.

The set of feasible incentive functions for principal j is therefore defined as $\mathcal{A}_j \equiv \{a_j \in A_j \colon a_j \text{ is } \mathcal{X}_j\text{-measurable}\}$. $\mathcal{A}_j$ differs in the part of the agent's action that is contractible between principal j and the agent. Let $\mathcal{A} \equiv \times_{k=1}^{J} \mathcal{A}_k$. In public common agency, principal j can make his action contingent on the whole action taken by the agent. In this case, each action x is an equivalence class. In private common agency, the agent's action x is decomposed into J components, $x = [x_1, \ldots, x_J]$, and principal j can make his action contingent on only the jth component, x_j, of the agent action. In this case, an equivalence class is the set of the agent's actions that have the same jth component.[5]

A principal may offer to the agent a mechanism that assigns an incentive function as a function of the agent's message. By doing so, he can make his incentive function itself responsive to the agent's report on both her type and what the other principals are doing in the market. Formally, a mechanism that principal j offers is a measurable mapping $\gamma_j \colon M_j \to \mathcal{A}_j$, where M_j is a set of messages available for the agent. When the agent sends a message $m_j \in M_j$ to principal j, the incentive function $\gamma_j(m_j) \in \mathcal{A}_j$ is assigned. Let Γ_j be the set of mechanisms available for principal j and $\Gamma \equiv \times_{k=1}^{J} \Gamma_k$. The common agency game relative to Γ starts when each principal j simultaneously offers a mechanism from Γ_j. After seeing a profile of mechanisms $\gamma = [\gamma_1, \cdots, \gamma_I] \in \Gamma$, the agent sends messages to those principals and takes an action from X. The agent's action then determines principals' actions given the incentive functions specified by the agent's messages. Finally, payoffs are realized.

The agent's continuation strategy is characterized by a measurable mapping $\delta_0 \colon \Gamma \times \Omega \to \Delta(M \times X)$, where $M \equiv \times_{k=1}^{J} M_k$. Given a continuation strategy δ_0, let $\delta_0(\gamma, \omega)$ denote a probability distribution on the agent's action and messages. Any continuation strategy δ_0 can be decomposed into $\delta_0^x \colon \Gamma \times \Omega \to \Delta(X)$ and $\delta_0^m \colon X \times \Gamma \times \Omega \to \Delta(M)$. Let $\delta_0^x(\gamma, \omega)$ denote

[5] Martimort (2007) coined the terminologies of public common agency and private common agency. In the example of private common agency where the buyer (agent) buys a good from several sellers (principals), the jth component of the agent' action can be interpreted as the quantity of the good that the buyer buys from seller j.

the probability distribution on the agent's action conditional on (γ, ω) and $\delta_0^m(x, \gamma, \omega)$ the probability distribution on the agent's messages across principals conditional on (x, γ, ω). When the agent's continuation strategy is δ_0, her continuation payoff is

$$U(\gamma, \delta_0, \omega) \equiv \int_X \left[\int_M u\left(\gamma_1(m_1)(x), \ldots, \gamma_J(m_J)(x), x, \omega\right) d\delta_0^m(x, \gamma, \omega) \right] d\delta_0^x(\gamma, \omega)$$

at each $(\gamma, \omega) \in \Gamma \times \Omega$. Let C be the set of all continuation strategies for the agent. A continuation strategy δ_0 is a continuation equilibrium relative to Γ if $U(\gamma, \delta_0, \omega) \geq U(\gamma, \delta_0', \omega)$ for all $(\gamma, \delta_0', \omega) \in \Gamma \times C \times \Omega$. Let $\mathcal{C}$ be the set of all continuation equilibria relative to Γ. Given $\delta_0 \in \mathcal{C}$, principal j's continuation payoff is

$$V_j(\gamma, \delta_0) \equiv \int_\Omega \left[\int_X \left(\int_M v_j\left(\gamma_1(m_1)(x), \ldots, \gamma_J(m_J)(x), x, \omega\right) d\delta_0^m(x, \gamma, \omega) \right) d\delta_0^x(\gamma, \omega) \right] dF$$

at each $\gamma \in \Gamma$. Let $\delta_j \in \Delta(\Gamma_j)$ be principal j's strategy. Given a continuation equilibrium δ_0 and the other principals' strategies δ_{-j}, principal j's payoff associated with δ_j is

$$V_j(\delta_j, \delta_{-j}, \delta_0) \equiv \int_{\Gamma_j} \left(\int_{\Gamma_{-j}} V_j(\gamma_j, \gamma_{-j}, \delta_0) d\delta_{-j} \right) d\delta_j.$$

A strategy profile $\delta = [\delta_1, \ldots, \delta_J, \delta_0]$ is a *perfect Bayesian equilibrium* (henceforth simply an equilibrium) relative to Γ if (i) $\delta_0 \in \mathcal{C}$ and (ii) for all $j \in \mathcal{J}$ and all $\delta_j' \in \Delta(\Gamma_j)$, $V_j(\delta_j, \delta_{-j}, \delta_0) \geq V_j(\delta_j', \delta_{-j}, \delta_0)$.

We will denote the set of equilibria relative to Γ by $E(\Gamma)$. For any equilibrium $\delta \in E(\Gamma)$, we will then denote by $\pi_\delta \colon \Omega \to \Delta(X \times Y)$ the associated social choice function, which characterizes the equilibrium allocation in $\delta \in E(\Gamma)$.

3 Competition relative to $\mathcal{A}$

In the common agency game relative to $\mathcal{A}$, each principal simultaneously offers an incentive function to the agent and then the agent takes her action, which subsequently determines each principal's action given his incentive function. Let $\sigma_0 : \mathcal{A} \times \Omega \to \Delta(X)$ denote the agent's action strategy in the common agency game relative to $\mathcal{A}$. Let $\mathcal{Z}$ be the set of all continuation equilibria relative to $\mathcal{A}$. Given a continuation equilibrium $\sigma_0 \in \mathcal{Z}$, we can define the agent's continuation payoff $U(a, \sigma_0, \omega)$ for all $(a, \omega) \in \mathcal{A} \times \Omega$. Let $\sigma_j \in \Delta(\mathcal{A}_j)$ denote principal j's strategy. Given an equilibrium $\sigma = [\sigma_1, \ldots, \sigma_J, \sigma_0]$ relative to $\mathcal{A}$, we can define principal j's equilibrium payoff $V_j(\sigma_j, \sigma_{-j}, \sigma_0)$.

An incentive function a_j can be thought of as a mechanism $\gamma_j^a \colon M_j^a \to \mathcal{A}_j$, where the set of messages, M_j^a, is a singleton. Let Γ_j^a be the set of all mechanisms given M_j^a. Let $\Gamma^a \equiv \times_{k=1}^{J} \Gamma_j^a$. Being consistent with the definition of an enlargement of menus in Pavan and Calzolari (2010), we define an enlargement of incentive functions as follows: $\Gamma \equiv \times_{k=1}^{J} \Gamma_k$ is an enlargement of Γ^a (or equivalently an enlargement of $\mathcal{A}$) if, for all $j \in \mathcal{J}$, (i) there exists an embedding $\phi_j \colon \Gamma_j^a \to \Gamma_j$;[6] and (ii) for any $\gamma_j \in \Gamma_j$, $\gamma_j(M_j)$, the image set of γ_j, is compact. Our interest is when the set of equilibrium allocations relative to any enlargement Γ of $\mathcal{A}$ is the same as the set of equilibrium allocations relative to $\mathcal{A}$. To this end, let us introduce a strict monotonicity condition for the agent's preference relation with respect to each principal's action.

Definition 1 *The agent's preference relation is* strictly monotone *in each principal's action if, for each $j \in \mathcal{J}$, each equivalence class $\tilde{x} \in \mathcal{X}_j$, and each compact subset $B_j^* \subset Y_j$,*

$$\arg\max_{y_j \in B_j^*} \; u(y_j, y_{-j}, x, \omega)$$

is the same for all $(y_{-j}, x, \omega) \in Y_{-j} \times \tilde{x} \times \Omega$ and it is a singleton.

For any $j \in \mathcal{J}$, any $\gamma_j \in \Gamma_j$, and any $\tilde{x} \in \mathcal{X}_j$ define $B_j(\tilde{x}, \gamma_j)$ as

$$B_j(\tilde{x}, \gamma_j) \equiv \{y_j \in Y_j \colon y_j = \gamma_j(m_j)(x) \text{ for all } m_j \in M_j \text{ and some } x \in \tilde{x}\}. \tag{1}$$

$\gamma_j(m_j)$ is the incentive function that principal j assigns when the agent sends a message m_j so that $B_j(\tilde{x}, \gamma_j)$ is the set of principal j's actions that the agent can induce when she takes any action x in an equivalence class $\tilde{x}$.[7] For all $j \in \mathcal{J}$, all $\gamma_j \in \Gamma_j$, and all $x \in X$, let

$$\psi_j(\gamma_j)(x) \equiv \arg\max_{y_j \in B_j(\tilde{x}, \gamma_j)} \; u(y_j, y_{-j}, x, \omega) \tag{2}$$

[6] An embedding $\phi_j \colon \Gamma_j^a \to \Gamma_j$ can be thought of as an injective function such that for all $\gamma_j^a \in \Gamma_j^a$, γ_j^a and $\phi_j(\gamma_j^a)$ have the same image set.

[7] If x and x' belong to the same equivalence class $\tilde{x}$, then $\gamma_j(m_j)(x) = \gamma_j(m_j)(x')$ for any $m_j \in M_j$. Therefore, the set on the right hand side of (1) is for all $x \in \tilde{x}$.

be principal j's action that maximizes the agent's payoff among all actions in $B_j(\tilde{x}, \gamma_j)$ when she takes x, where $\tilde{x}$ in (2) is the equivalence class that satisfies $x \in \tilde{x}$ given x. When the agent's preference relation is strictly monotone in each principal's action, $\psi_j(\gamma_j)(x)$ is a singleton for all $x \in X$ so that $\psi_j(\gamma_j)$ itself becomes an incentive *function* that specifies principal j's action as a function of the agent's action.

For technical simplicity, we assume that $\psi_j(\gamma_j)(x)$ is non-empty for all $j \in \mathcal{J}$, all $x \in X$, and all $\gamma_j \in \Gamma_j$

Lemma 1 *Suppose that the agent's preference relation is strictly monotone in each principal's action. In any continuation equilibrium δ_0 relative to Γ, any $(m_1, \ldots, m_J)$ in the support of $\delta_0^m(x, \gamma, \omega)$ satisfies*

$$\gamma_j(m_j)(x) = \psi_j(\gamma_j)(x) \tag{3}$$

for all $(x, \gamma, \omega) \in X \times \Gamma \times \Omega$ and all $j \in \mathcal{J}$.

Proof. Let $\gamma = [\gamma_1, \ldots, \gamma_J]$ be the profile of mechanisms that principals offer. Given a continuation equilibrium δ_0 relative to Γ, let $\delta_0^{m_j}(x, \gamma, \omega)$ be the marginal probability distribution on M_j conditional on (x, γ, ω). Suppose that the agent's preference relation is strictly monotone in each principal's action. Given γ_j, the set of principal j's actions, $B_j(\tilde{x}, \gamma_j)$ defined in (1), that the agent can induce depends on the action x that she takes because x subsequently determines $\tilde{x}$. Once $B_j(\tilde{x}, \gamma_j)$ is determined, the agent will always choose a message that leads to $\psi_j(\gamma_j)(x)$ in $B_j(\tilde{x}, \gamma_j)$ because of the strict monotonicity of the agent's preference relation. It implies that given x and γ_j, any m_j in the support of $\delta_0^{m_j}(x, \gamma_j, \gamma_{-j}, \omega)$ must satisfy (3) regardless of γ_{-j} and ω. Therefore, any $(m_1, \ldots, m_J)$ in the support of $\delta_0^m(x, \gamma, \omega)$ satisfies (3) for all $(x, \gamma, \omega) \in X \times \Gamma \times \Omega$ and all $j \in \mathcal{J}$. ■

Lemma 1 leads to the main result of the paper below.

Theorem 1 *Suppose that the agent's preference relation is strictly monotone in each principal's action. Then, a social choice function $f \colon \Omega \to \Delta(X \times Y)$ is supported by an equilibrium relative to $\mathcal{A}$ if and only if it is supported by an equilibrium relative to any enlargement Γ of $\mathcal{A}$.*

The intuition of Theorem 1 can be explained in an example with two principals and one agent. Let $\Omega = \{\omega\}$. First suppose that a social choice function f is supported by an equilibrium $[\delta_1, \delta_2, \delta_0]$ relative to Γ so that $f = \pi_\delta$. With a slight abuse of notation, let $\delta_j(\gamma_j)$ be the probability that principal j offers a mechanism γ_j in Γ_j and $\delta_0^x(x|\gamma,\omega)$ the probability that the agent takes an action x conditional on (γ,ω) given her continuation strategy δ_0. Let the equilibrium strategies for principals $[\delta_1,\delta_2]$ be $\delta_1(\gamma_1)=1$ and $\delta_2(\gamma_2)=\delta_2(\gamma_2')=\delta_2(\gamma_2'')=1/3$. Suppose that, on the equilibrium path, the marginal probability distribution $\delta_0^x(\gamma,\omega)$ on the agent's actions satisfies:

	γ_2	γ_2'	γ_2''
γ_1	$\delta_0^x(x\|\gamma_1,\gamma_2,\omega)=1/2$ $\delta_0^x(x'\|\gamma_1,\gamma_2,\omega)=1/2$	$\delta_0^x(x\|\gamma_1,\gamma_2',\omega)=1$	$\delta_0^x(x''\|\gamma_1,\gamma_2'',\omega)=1$

We need to convert the equilibrium $[\delta_1,\delta_2,\delta_0]$ relative to Γ to an equilibrium $[\sigma_1,\sigma_2,\sigma_0]$ relative to $\mathcal{A}$ that induces $\pi_\sigma=\pi_\delta$. Assume that the agent's preference relation is strictly monotone in each principal's action. Then, lemma 1 shows that principal j's action, $\psi_j(\gamma_j)(x)$, is uniquely determined by the agent's action x given γ_j in any continuation equilibrium.

Let $\psi_1(\gamma_1)=a_1$. If principal 1 offers γ_1 and the agent takes x, then his action becomes $\psi_1(\gamma_1)(x)=a_1(x)$ regardless of the agent's type and the actions taken by the other principals. Therefore, principal 1's action can be preserved by directly offering a_1 instead of γ_1 so that let principal 1 choose his strategy σ_1 with

$$\sigma_1(a_1)=\delta_1(\gamma_1)=1 \tag{4}$$

in the game relative to $\mathcal{A}$. Suppose that $\psi_2(\gamma_2)=\psi_2(\gamma_2')=a_2$ and $\psi_2(\gamma_2'')=a_2''$ with $a_2\neq a_2''$. Note that both γ_2 and γ_2' are converted into a_2 but γ_2'' into a_2'' so that let principal 2 choose his strategy σ_2 with

$$\sigma_2(a_2) = \delta_2(\gamma_2)+\delta_2(\gamma_2')=1/3+1/3=2/3, \tag{5}$$
$$\sigma_2(a_2'') = \delta_2(\gamma_2'')=1/3. \tag{6}$$

Suppose that principal 1 offers a_1 and principal 2 offers a_2 in the game relative to $\mathcal{A}$. It implies that principal 1 offers γ_1 and principal 2 offers γ_2 or γ_2' in the game relative to Γ. Because $\delta_1(\gamma_1)=1$ and $\delta_2(\gamma_2)=\delta_2(\gamma_2')=1/3$, it is clear that (γ_1,γ_2) is realized with probability 1/2 and (γ_1,γ_2') is realized with probability 1/2 given that principal 1 offers γ_1 and principal 2 offers γ_2 or γ_2' in the game relative to Γ. Subsequently, when principal 1 offers a_1 and principal 2 offers a_2, we can assign the distribution on the agent's actions $\sigma_0(a_1,a_2,\omega)$ with

$$\sigma_0(x|a_1,a_2,\omega) = 1/2\times\delta_0^x(x|\gamma_1,\gamma_2,\omega)+1/2\times\delta_0^x(x|\gamma_1,\gamma_2',\omega)=3/4 \tag{7}$$
$$\sigma_0(x'|a_1,a_2,\omega) = 1/2\times\delta_0^x(x'|\gamma_1,\gamma_2,\omega)=1/4. \tag{8}$$

When (a_1, a_2) is a profile of incentive functions offered to the agent, it is optimal for her to take her action according to $\sigma_0(a_1, a_2, \omega)$ specified in (7) and (8) because it is optimal for her to take any action in the support of $\delta_0^x(\gamma_1, \gamma_2, \omega)$ or the support of $\delta_0^x(\gamma_1', \gamma_2, \omega)$. Similarly, when principal 1 offers a_1 and principal 2 offers a_2'', we can assign the agent's optimal distribution on her actions $\sigma_0(a_1, a_2'', \omega)$ with

$$\sigma_0(x''|a_1, a_2'', \omega) = \delta_0^x(x''|\gamma_1, \gamma_2'', \omega) = 1. \tag{9}$$

Note that $\sigma = [\sigma_1, \sigma_2, \sigma_0]$ specified in (4) - (9) preserves the probability distribution on the agent's actions and the profiles of incentive functions that are induced by principals' mechanisms on the equilibrium path: i.e., (x, a_1, a_2) with prob. 1/2, (x', a_1, a_2) with prob. 1/6, and (x'', a_1, a_2'') with prob. 1/3. Consequently it implies that π_δ is reproduced by π_σ with σ specified in (4) - (9) because, given the strict monotonicity of the agent's preference relation, principal j's action $\psi_j(\gamma_j)(x)$ is uniquely determined by the agent's action x given γ_j in any continuation equilibrium. As shown in the proof, we can also properly assign a continuation equilibrium upon each principal j's deviation in the game relative to $\mathcal{A}$ that prevents him from deviating to any incentive function.

It is simpler to prove the other way around. Suppose that a social choice function f is supported by an equilibrium σ relative to $\mathcal{A}$, i.e. $f = \pi_\sigma$. Any incentive function a_k can be viewed as a mechanism γ_k that assigns a_k regardless of the agent's message and hence we only need to consider principal j's deviation to a mechanism that does not assign the same incentive function for all messages. When principal j deviates to such a mechanism, say γ_j, the agent chooses her action x according to $\sigma_0(\psi_j(\gamma_j), a_{-j}, \omega)$ and send to principal j a message m_j that satisfies $\gamma_j(m_j)(x) = \psi_j(\gamma_j)(x)$ given her action x. Then, principal j's payoff upon deviation to γ_j is the same as the one he receives when he deviates to the corresponding incentive function $\psi_j(\gamma_j)$ in the game relative to $\mathcal{A}$ so that it is not a profitable deviation relative to Γ.

4 Discussion

We discuss how our result is related to the "no externalities" condition in Peters (2003) and the difference between the incentive function and the menu.

4.1 No Externalities Conditions

Peters (2003) focuses on pure-strategy equilibria where principals employ pure strategies. Theorem 2 in Peters (2003) shows that any pure-strategy equilibrium relative to type direct mechanisms that assign an incentive function contingent on the agent's type report continues to be an equilibrium relative to menus (or equivalently any complex mechanisms) while the competition relative to type direct mechanisms may not generate all equilibrium allocations relative to any complex mechanisms.

Peters then defines a "*no externalities*" condition for each principal's action. Theorem 4 in Peters (2003) showed that if the "no externalities" condition holds, payoffs associated with any pure-strategy equilibrium relative to any complex mechanisms that assign an incentive function in $\mathcal{A}$ contingent on the agent's message are preserved by a pure-strategy equilibrium relative to random incentive functions. Principal j's random incentive function specifies a probability distribution on y_j contingent on the part of agent's action that is contractible while an incentive function in $\mathcal{A}$ specifies y_j directly.[8] The "no externalities" condition in Peters (2003, 2007) is stated as follows:

D1. For each $j \in \mathcal{J}$, there exists a function $\bar{v}_j : Y_j \times X \times \Omega \to \mathbb{R}$ such that for all $(y_1, \ldots, y_J) \in \times_{k=1}^{J} Y_k$, all $x \in X$, and all $\omega \in \Omega$

$$v_j(y_1, \ldots, y_J, x, \omega) = \bar{v}_j(y_j, x, \omega)$$

D2. For each $j \in \mathcal{J}$, each $\tilde{x} \in \mathcal{X}_j$, each closed subset $B_j^* \subset Y_j$, the set

$$\left\{ y_j \in B_j^* : u(y_j, y_{-j}, x, \omega) \geq u(y_j', y_{-j}, x, \omega) \text{ for all } y_j' \in B_j^* \right\} \quad (10)$$

is the same for all $(y_{-j}, x, \omega) \in Y_{-j} \times \tilde{x} \times \Omega$.

The strict monotonicity of the agent's preference relation satisfies condition D2 in a stronger form in that it additionally requires that the set in (10) be a *singleton*. The strict monotonicity of the agent's preference relation implies that even when the agent randomizes her communication with principal j in the game relative to Γ, principal j's action will be always the same given the agent's type and her action. Not only does it make random incentive functions superfluous, contrary to Theorem 4 in Peters (2003), but it also makes it possible for any mechanism to be associated

[8] A random incentive function offered by principal j is defined as $\alpha_j : X \to \Delta(Y_j)$. While an incentive contract is defined as $a_j : X \to Y_j$.

with a unique incentive function: Furthermore, restrictions on principals' preference relations such as condition D1 are not required to establish the result in Theorem 1 in our paper.

Peters's results show how to preserve players' *equilibrium payoffs* in pure strategy equilibria with the "no externalities" condition. With the strict monotonicity of the agent's preference relation only, we strengthen Peters's results by allowing for all equilibria including mixed-strategy equilibria and by preserving *equilibrium allocations*: Theorem 1 shows that the set of *all equilibrium allocations* relative to any enlargement Γ of $\mathcal{A}$ is the same as the set of all equilibrium allocations relative to $\mathcal{A}$.

The strict monotonicity of the agent's preference relation is stronger than the "no externalities" condition for the agent's preference relation. However, it does not require that the agent's payoff function be quasi-linear or additively separable (or separable a la Attar et al. 2008). The strict monotonicity is sufficiently general to allow the agent's preferences over her actions to depend on principals' actions and the agent's type in public common agency and to allow the agent's preferences over x_j to depend on principals' actions, her choice of x_{-j}, and her type in private common agency.

Notably, the strict monotonicity of the agent's preference relation is satisfied when each principal's action can be characterized by monetary transfer such as monetary payment, price, tax rate, and etc. Let y_j be the amount of monetary transfer from the agent to principal j. Consider private common agency. Because only the jth component x_j of the agent's action is contractible, an equivalence class $\tilde{x}$ associated with x_j includes all actions with the jth component being x_j. Given the equivalence class $\tilde{x}$ associated with x_j, we have that

$$y_j < y_j' \Longrightarrow u(y_j, y_{-j}, x_j, x_{-j}, \omega) > u(y_j', y_{-j}, x_j, x_{-j}, \omega) \tag{11}$$

for all $(y_{-j}, x_{-j}, \omega) \in Y_{-j} \times X_{-j} \times \Omega$ because the agent always prefers less monetary transfer to principal j *given* x_j regardless of (y_{-j}, x_{-j}, ω). However, (11) allows the agent's preferences on x_j to depend on $(y_1, \ldots, y_J, x_{-j}, \omega)$. Consider public common agency. Because the whole action of the agent is contractible, each action x is an equivalence class so that each equivalence class is a singleton. Given each x, we have that

$$y_j < y_j' \Longrightarrow u(y_j, y_{-j}, x, \omega) > u(y_j', y_{-j}, x, \omega) \tag{12}$$

for all $(y_{-j}, \omega) \in Y_{-j} \times \Omega$ because the agent always prefers less monetary transfer to principal j *given* x regardless of (y_{-j}, ω). However, (12) allows the agent's preferences on x to depend on $(y_1, \ldots, y_J, \omega)$. It is clear that (11) and (12) satisfy the strict monotonicity of the agent's preference relation.

4.2 Incentive functions vs. Menus

A menu is an arbitrary set of alternatives from which the agent simply choose an alternative that she wants. The definition of the menu depends on how to view a mechanism. We take Peters's viewpoint in which a mechanism is a negotiation scheme that assigns an incentive function contingent on the agent's message. In this case, a menu is naturally defined as an arbitrary set of incentive functions from which the agent chooses an incentive function that she wants. Given this definition of the menu, the set of all incentive functions is strictly smaller than (i.e., a strict subset of) the set of all menus because an incentive function is a menu that includes only one incentive function.

Alternatively, one can view a mechanism as a negotiation scheme that directly assigns a pair of the principal's action and the contractible part of the agent's action contingent on the agent's message. In this case, a menu can be defined as an arbitrary set of pairs of the principal's action and the contractible part of the agent's action. Even with this alternative definition, the menu is not equivalent to the incentive function so that the set of incentive contracts is strictly smaller than the set of menus. This point can be explained simply with a single principal and a single agent: The principal takes an action y from a set Y and the agent takes an action x from a set X. For simplicity, assume that the whole action x is contractible so that each single action is an equivalence class. In the alternative viewpoint of a mechanism, a mechanism is a mapping from M, the set of messages, into $X \times Y$. Because of the standard revelation principle, we only need to consider incentive-compatible type direct mechanisms. It is quite straightforward to show that any incentive-compatible type direct mechanism can be replaced with a menu.

Consider the following example in which $Y = \{y^\circ, y'\}$, $X = \{x^\circ, x'\}$, and $\Omega = \{\omega^\circ, \omega', \omega''\}$. Suppose that the agent's payoffs are characterized as follows:

	ω°	ω'	ω''
(y°, x°)	1	2	3
(y', x°)	4	1	4
(y°, x')	3	3	2
(y', x')	2	4	1

Let $\gamma_D = \{\mathbf{x}, \mathbf{y}\}$ be a type direct mechanism, where $\mathbf{x}\colon \Omega \to X$ specifies the agent's action as a function of her type report and $\mathbf{y}\colon \Omega \to Y$ specifies the principal's action as a function of her type report. Given the agent' payoff structure above, the following type direct mechanism $\gamma_D = \{\mathbf{x}, \mathbf{y}\}$ is incentive compatible:

$$(\mathbf{x}(\omega^\circ), \mathbf{y}(\omega^\circ)) = (y^\circ, x') \tag{13}$$
$$(\mathbf{x}(\omega'), \mathbf{y}(\omega')) = (y', x') \tag{14}$$
$$(\mathbf{x}(\omega''), \mathbf{y}(\omega'')) = (y^\circ, x^\circ) \tag{15}$$

Instead of offering the type direct mechanism $\gamma_D = \{\mathbf{x}, \mathbf{y}\}$ satisfying (13) - (15), the principal can simply offer to the agent the corresponding menu that is the image set of the incentive-compatible type direct mechanism:

$$\gamma^* = \{(\mathbf{x}(\omega), \mathbf{y}(\omega))\colon \omega \in \Omega\} = \{(y^\circ, x'), (y'x'), (y^\circ, x^\circ)\}.$$

Therefore, the principal can rely on menus instead of incentive compatible direct mechanisms.

While every incentive-compatible type direct mechanism can be converted into a menu by taking its image set, not every incentive-compatible direct mechanism, or equivalently not every menu, can be converted into an incentive function that specifies the principal's action a *function* of the agent's action. For example, when the agent takes x', any incentive function can assign only a single action for the principal because it is a function. However, the type direct mechanism $\gamma_D = \{\mathbf{x}, \mathbf{y}\}$ satisfying (13) - (15) or the corresponding menu γ^* allows the agent to choose either y° or y' given her action choice x'. Therefore, the type direct mechanism $\gamma_D = \{\mathbf{x}, \mathbf{y}\}$ satisfying (13) - (15) or the corresponding menu γ^* cannot be converted to any incentive function that specifies the principal's action as a function of the agent's action.

Given the alternative definition of the menu, any incentive function $a\colon X \to Y$ can be converted into a menu by taking its graph $\{(y, x) : y = a(x)\ \forall x \in X\}$. Such a menu does not includes two different pairs (y, x) and (y', x') with $y \neq y'$ and $x = x'$ because it is converted from an incentive function. Let Γ^* be the set of all possible menus given the alternative definition of the menu. Let $\tilde{\Gamma}$ be the set of menus that are converted from all incentive functions by taking their graphs. Given the alternative definition of the menu, $\tilde{\Gamma}$ is strictly smaller than (i.e., a strict subset of) Γ^*.

When the agent's action is fully contractible in the case with a single principal and a single agent, the strict monotonicity of the agent's preference relation can be expressed as follows: For each $j \in \mathcal{J}$, each $x \in X$, each compact subset $B^* \subset Y$,

$$\arg\max_{y \in B^*} \; u(y, x, \omega) \tag{16}$$

is the same for all $\omega \in \Omega$ and it is a singleton. When the agent's preference relation is strictly monotone in the principal's action, we have that, for any incentive-compatible type direct mechanism $\gamma_D = \{\mathbf{x}, \mathbf{y}\}$,

$$\mathbf{x}(\omega) = \mathbf{x}(\omega') \Rightarrow \mathbf{y}(\omega) = \mathbf{y}(\omega') \tag{17}$$

for all $\omega, \omega' \in \Omega$.[9] (17) enables us to convert any incentive-compatible type direct mechanism $\gamma_D = \{\mathbf{x}, \mathbf{y}\}$ to an incentive function $a\colon X \to Y$ such that $a(\mathbf{x}(\omega)) = \mathbf{y}(\omega)$ for all $\omega \in \Omega$. The strict monotonicity of the agent's preference relation in common agency can be viewed as the generalization of (16).

5 Conclusion

Given any viewpoint of a mechanism, the principal can rely on the corresponding set of menus without loss of generality in the case with a single principal and a single agent. It is extended to the menu theorem for common agency (Peters 2001; Martimort and Stole 2002). Given the viewpoint that a mechanism is a negotiation scheme that assigns an incentive function contingent on the agent's message, a menu in Peters (2001, 2003) is defined as an arbitrary set of incentive functions. One can view a mechanism as a negotiation scheme that directly assigns a pair of the principal's action and the contractible part of the agent's action contingent on the agent's message: In this case, a menu is then an arbitrary set of y_j-x_j pairs, for example, in private common agency.[10]

[9] We can show this by contradiction. In contrary, suppose that for some $\omega, \omega' \in \Omega$, $\mathbf{x}(\omega) = \mathbf{x}(\omega')$ but $\mathbf{y}(\omega) \neq \mathbf{y}(\omega')$. The strict monotonicity and the incentive compatibility imply that the agent of type ω strictly prefers $\mathbf{y}(\omega)$ to $\mathbf{y}(\omega')$ given $\mathbf{x}(\omega)$. Because of the strict monotonicity, it means that the agent of type ω' also strictly prefers $\mathbf{y}(\omega)$ to $\mathbf{y}(\omega')$ given her same action choice $\mathbf{x}(\omega') = \mathbf{x}(\omega)$. Hence it contradicts the incentive compatibility.

[10] Again, it is important to know that the menu in this definition is also not equivalent to the incentive function because the incentive function assigns a unique y_j for any given x_j while the menu can assign many different y_j for any given x_j. In fact, the menu is equivalent to the incentive correspondence that assigns a set of principal j's actions for any given x_j.

Pavan and Calzolari (2010) however pointed out that the menu theorem provides no indication as to which incentive function or action pair the agent selects from the menus, nor does it permit one to restrict attention to a particular set of menus. In contrary, the taxation principle (Rochet 1985; Guesnerie 1995) shows that, for the case with a single principal and a single agent (or a continuum of agents), one can focus only on the set of incentive functions when monetary transfer (e.g., monetary payment, price, tax, and etc.) is involved.[11] The set of incentive functions is a strict subset of the set of all possible menus given any definition of the menu. By restricting to this particular subset of menus (i.e., the set of incentive functions), the taxation principle makes it possible to conveniently use Optimal Control Theory in deriving the equilibrium incentive function (Rochet 1985). Such a technique has been extended to common agency problems in deriving equilibrium incentive functions and allocations when the competition among principals are modeled relative to incentive functions (Martimort 2007). Our paper shows that, in addition to its tractability in deriving equilibrium allocations, the competition relative to incentive functions generates the set of equilibrium allocations that is the same as the set of all equilibrium allocations relative to any complex mechanisms when the agent's preference relation is strictly monotone in each principal's action. This result can be viewed as a *generalized taxation principle*, which provides the sufficient condition under which the set of all equilibrium allocations relative to incentive functions is the same as the set of all equilibrium allocations relative to any complex mechanisms in the environments with an arbitrary number of principals.

APPENDIX: Proof of Theorem 1

Suppose that a social choice function $f\colon \Omega \to \Delta(X \times Y)$ is supported by an equilibrium δ relative to Γ for any enlargement Γ of $\mathcal{A}$, i.e., $f = \pi_\delta$. For each $j \in \mathcal{J}$, let σ_j be the measure induced by δ_j through the map ψ_j. Let ψ_j^{-1} be the inverse correspondence of ψ_j. For any $a_j \in A_j$, define the set $D_j(a_j) \subset \Gamma_j$ as

$$D_j(a_j) \equiv \begin{cases} \psi_j^{-1}(a_j) \cap \text{ supp } \delta_j & \text{if } \psi_j^{-1}(a_j) \cap \text{ supp } \delta_j \neq \varnothing \\ \bar{\psi}_j^{-1}(a_j) & \text{otherwise,} \end{cases}$$

where $\bar{\psi}_j^{-1}(a_j)$ is an arbitrary mechanism in $\psi_j^{-1}(a_j)$. For any $a = [a_1, \ldots, a_J] \in \mathcal{A}$, let $D(a) = \times_{k=1}^{J} D_k(a_k) \subset \Gamma$.

[11] In this case, an incentive function is called a taxation schedule or nonlinear price.

From the equilibrium strategy profile δ relative to Γ, we can derive a joint probability distribution $b(D,\omega)$ on $M \times X$ for all $D \subset \Gamma$ and all $\omega \in \Omega$. Let $b^m(x,D,\omega)$ be the probability distribution on M conditional on (x,D,ω) that $b(D,\omega)$ induces. Let $b^x(D,\omega)$ be the marginal probability distribution on X that $b(D,\omega)$ induces. Construct the agent's continuation strategy $\sigma_0 : \mathcal{A} \times \Omega \to \Delta(X)$ relative to $\mathcal{A}$ as

$$\sigma_0(a,\omega) = b^x(D(a),\omega) \tag{18}$$

for all $(a,\omega) \in \mathcal{A} \times \Omega$. Lemma 1 implies that any $(m_1,\ldots,m_J)$ in the support of $b^m(x,D(a),\omega)$ induces the same profile of principals' actions, $[a_1(x),\ldots,a_J(x)]$. Therefore, (18) ensures that the social choice function $\pi_\delta\colon \Omega \to \Delta(X \times Y)$ associated with δ is the same as the social choice function $\pi_\sigma\colon \Omega \to \Delta(X \times Y)$ associated with σ given that σ_j is the measure induced by δ_j through the map ψ_j for each $j \in \mathcal{J}$.

When the agent chooses x, it is always optimal for her to report to principals messages that lead to $[a_1(x),\ldots,a_J(x)]$ given their mechanisms $\gamma \in D(a)$ according to Lemma 1. Given $\gamma \in D(a)$ and her payoff type $\omega \in \Omega$, the agent's optimal choice of his action then satisfies

$$x \in \arg\max_{x' \in X} u\left(a_1(x'),\ldots,a_1(x'),x',\omega\right). \tag{19}$$

Any x in the support of $b^x(D(a),\omega)$ satisfies (19) because the joint probability distribution $b(D(a),\omega)$ is derived from the continuation equilibrium δ_0 relative to Γ. Therefore, (18) implies that σ_0 is a continuation equilibrium relative to $\mathcal{A}$.

We only need to show that σ_j is a best response for principal j given σ_{-j}. Consider each principal j's payoff. For any $(a_j,a_{-j}) \in \mathcal{A}$, let

$$\begin{aligned} v_j^*(a_j,a_{-j}) &= \int_\Omega \left(\int_X v_j\left(a_j(x),a_{-j}(x),x,\omega,\right) db^x(D(a),\omega) \right) dF \\ &= \mathbb{E}_{\gamma \in D(a)} \left[\int_\Omega \left(\int_X v_j\left(a_j(x),a_{-j}(x),x,\omega\right) d\delta_0^x(\gamma,\omega) \right) dF \right]. \end{aligned}$$

Integrating $v_j^*(a_j,a_{-j})$ using σ_{-j} yields

$$\begin{aligned} & V_j(a_j,\sigma_{-j},\sigma_0) \qquad (20) \\ =\; & \mathbb{E}_{\gamma_j \in D_j(a_j)} \left[\int_{\Gamma_{-j}} \left\{ \int_\Omega \left(\int_X v_j\left(a_j(x),a_{-j}(x),x,\omega\right) d\delta_0^x(\gamma_j,\gamma_{-j},\omega) \right) dF \right\} d\delta_{-j} \right] \\ =\; & \mathbb{E}_{\gamma_j \in D_j(a_j)}[V_j(\gamma_j,\delta_{-j},\delta_0)]. \end{aligned}$$

Now we like to prove that σ_j is a best reply for principal j to σ_{-j} given the construction of σ through $\psi = [\psi_1, \ldots, \psi_J]$ by the equilibrium δ. First, consider any $a_j \in$ supp σ_j. The construction of σ_j implies that $D_j(a_j) = \psi_j^{-1}(a_j) \cap$ supp $\delta_j \neq \varnothing$. Because any $\gamma_j \in D_j(a_j)$ is in the support of δ_j, (20) implies that, for all $\gamma_j \in D_j(a_j) = \psi_j^{-1}(a_j) \cap$ supp δ_j,

$$V_j(a_j, \sigma_{-j}, \sigma_0) = V_j(\gamma_j, \delta_{-j}, \delta_0) = V_j(\delta_j, \delta_{-j}, \delta_0) \tag{21}$$

Second, consider any $\hat{a}_j \notin$ supp σ_j. Then $\psi_j^{-1}(\hat{a}_j) \cap$ supp $\delta_j = \varnothing$. In this case, $D_j(\hat{a}_j)$ is a singleton of $\bar{\psi}_j^{-1}(\hat{a}_j) \in \Gamma_j$ and (20) implies that

$$V_j(\hat{a}_j, \sigma_{-j}, \sigma_0) = V_j(\bar{\psi}_j^{-1}(\hat{a}_j), \delta_{-j}, \delta_0) \tag{22}$$

(21) and (22) show that σ_j is a best response for principal j when the other principals use σ_{-j} given a continuation equilibrium σ_0. Therefore, the social choice function $f : \Omega \to \Delta(X \times Y)$ is also supported by an equilibrium relative to $\mathcal{A}$.

Now suppose that a social choice function $f : \Omega \to \Delta(X \times Y)$ is supported by an equilibrium σ relative to $\mathcal{A}$. Note that any incentive function a_k can be viewed as a mechanism γ_k that assigns a_k regardless of the agent's message. For principal j's deviation to mechanisms in Γ_j, one can associate σ_0, due to by Lemma 1, with a continuation equilibrium strategy $\tilde{\delta}_0 : \Gamma_j \times \mathcal{A}_{-j} \times \Omega \to \Delta(M_j \times X)$ relative to $\Gamma_j \times \mathcal{A}_{-j}$ as follows. The probability distribution $\tilde{\delta}_0^{m_j}(x, \gamma_j, a_{-j}, \omega)$ on M_j satisfies, for all $m_j \in$ supp $\tilde{\delta}_0^{m_j}(x, \gamma_j, a_{-j}, \omega)$,

$$\gamma_j(m_j)(x) = \psi_j(\gamma_j)(x) \tag{23}$$

and the probability distribution $\tilde{\delta}_0^{x}(\gamma_j, a_{-j}, \omega)$ on X satisfies

$$\tilde{\delta}_0^{x}(\gamma_j, a_{-j}, \omega) = \sigma_0(\psi_j(\gamma_j), a_{-j}, \omega). \tag{24}$$

If principal j deviates to a mechanism γ_j in Γ_j, his payoff becomes

$$V_j(\gamma_j, \sigma_{-j}, \tilde{\delta}_0) = V_j(\psi_j(\gamma_j), \sigma_{-j}, \sigma_0) \tag{25}$$

because of (24). Because σ is an equilibrium relative to $\mathcal{A}$ and $\psi_j(\gamma_j) \in \mathcal{A}_j$, we have

$$V_j(\sigma_j, \sigma_{-j}, \sigma_0) \geq V_j(\psi_j(\gamma_j), \sigma_{-j}, \sigma_0). \tag{26}$$

Combining (25) and (26) yields $V_j(\sigma_j, \sigma_{-j}, \sigma_0) \geq V_j(\gamma_j, \sigma_{-j}, \tilde{\delta}_0)$. Therefore, the social choice function $f : \Omega \to \Delta(X \times Y)$ can be also supported by an equilibrium relative to Γ.

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