Implicit Collusion in Non-Exclusive Contracting under Adverse Selection

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October 5, 2011

Abstract

This paper studies how implicit collusion may take place in non-exclusive contracting under adverse selection when multiple agents (e.g., entrepreneurs with risky projects) non-exclusively trade with multiple firms (e.g., banks). It introduces the notion of the dual-additive price schedule, which makes agents non-exclusively trade with multiple firms in the market without arbitrage opportunities. It then shows that any dual-additive price schedule can be supported as equilibrium terms of trade in the market if each firm's expected profit is no less than its reservation profit. Firms sustain collusive outcomes through triggering trading mechanisms in which they change their terms of trade contingent only on agents' reports on the lowest average price that the deviating firm's trading mechanism would induce.

1 Introduction

Trading in the decentralized markets is frequently non-exclusive by nature and involves asymmetric information between contracting parties. For example, a bank may lend money to many entrepreneurs who have private information on

*I thank Andrea Attar, Archishman Chakraborty, Maxim Ivanov, David Martimort, Jiwoong Lee, Gregory Pavlov, Mike Peters, Jeff Racine and the seminar audience at McMaster University, the 22nd ICGT in Stony Brook, NY and EEA-ESEM 2011 in Oslo, Norway for their comments: Especially Lee showed me how to simplify the proof of Corollary 1 and Ivanov provided comments on super-additive functions and the related examples, which are incorporated into the examples for the dual-additive functions. Address: Department of Economics, 1280 Main Street West, McMaster University, Hamilton, Ontario, Canada L8S 4M4; Email: hansj@mcmaster.ca

their risky projects and vice versa an entrepreneur may borrow money from many banks to finance his risky project. Various financial assets including derivatives are also non-exclusively traded among sellers and buyers.[1] Some traders tend to have more information on the underlying values of structured assets such as derivatives than others do.

Non-exclusive contracting with multiple agents (e.g., entrepreneurs in loan contracting) is a complex process for firms (e.g., banks in loan contracting) because agents can also contract with competing firms. In this contracting environment, agents may well communicate with firms at the contracting stage because firms can ask agents about competing firms' terms of trade (e.g. loan amount and interest pairs in loan contracting). Importantly, when multiple agents communicate with firms, firms can compare what agents are telling. This may make it easier for firms to acquire the true information on competing firms' terms of trade from multiple agents. Subsequently, they may want to offer trading mechanisms in which their terms of trade depend on agents' reports on competing firms' terms of trade. In this way, firms can actively punish a deviating firm by changing their terms of trade upon agents' reports on the deviating firm's terms of trade and hence they may sustain many collusive outcomes that are not possible when there is only one agent.

We assume that firms do not observe trading mechanisms offered by competing firms. Alternatively, firms do observe competing firms' trading mechanisms but they do not have the full commitment so that they cannot write binding contracts directly contingent on competing firms' offers that they observe. A firm can make quantity and monetary payment pairs across agents only contingent on agents' messages. This paper analyzes the degree of collusion among firms and the scope of collusive outcomes in non-exclusive contracting when multiple firms engage in communication with multiple agents in the contracting process.

Consider a market for a good where each privately-informed agent can trade with any number of firms and each firm can also trade with any number of privately-informed agents. Firms can freely offer any arbitrary trading mechanism that make quantity and monetary payment pairs across agents contingent

[1] The volume and variety of derivatives trading have grown dramatically. According to the Preliminary Staff Report of the Financial Crisis Inquiry Commission (2010), global over-the-counter derivatives grew seven-fold from $88 trillion in notional amount outstanding in 1999 to $615 trillion notional amount outstanding in 2009. To put it into perspective, the value of the world's financial assets, including all stock, bonds, and bank deposits, was $167 trillion in 2007. The counter-party credit risk is also highly concentrated. For example, 97% of derivatives on the books of U.S. banks are held by the largest five banks (JP Morgan Chase, Goldman Sachs, Bank of America, Citibank, and Wells Fargo).

on their messages.[2] A trading mechanism can be quite complex in the degree and nature of the communication that it permits regarding what competing firms are doing. Once agents observe trading mechanisms offered by firms, they communicate with firms by sending messages. A profile of messages that each firm receives then determines the quantity and monetary payment pairs, one for each agent.

We first examine the market terms of trade that make agents non-exclusively trade with firms without arbitrage opportunities. The market terms of trade are characterized by a price schedule $\mathbf{y}(x)$ when the agent can trade with each firm according to it. It specifies the agent's monetary payment as a function of quantity x. We introduce the property of a dual-additive price schedule. A price schedule is dual-additive if (1) $\mathbf{y}(x+x') \geq \mathbf{y}(x)+\mathbf{y}(x')$ when $x, x' \geq 0$ or $x, x' \leq 0$ and (2) $\mathbf{y}(x+x') \leq \mathbf{y}(x)+\mathbf{y}(x')$ when $x \geq 0$ and $x' \leq 0$.[3] Conditions 2 ensures no arbitrage opportunities because the agent cannot find a profitable trading opportunity in which he buys the good from a firm at a lower price and sell it to another firm at a higher price. Conditions 1 and 2 ensure that if the agent buys (sell) from (to) one firm, then it is optimal for him to buy (sell) from (to) every firm. Various price schedules satisfy the dual-additivity. For example, a linear price schedule (i.e, constant unit price) is dual-additive. A price schedule that induces non-decreasing average price with $\mathbf{y}(0) = 0$ is also dual-additive.[4] The property of dual-additivity is more general than the property of non-decreasing average price. In particular, it may admit decreasing average price over some range of the domain. When a price schedule is dual-additive, it ensures that each agent endogenously trades with all firms given a positive (negative) quantity that he wants to trade. It allows firms to

[2]McAfee (1993) and Peck (1994) pointed out that when multiple sellers compete, the standard revelation principle for the payoff type direct mechanism does not hold. The reason is that agents have private information on what's happening with competing sellers (e.g., their trading mechanisms, agents' communication with them, terms of trade, and etc.) on top of private information on their payoff types. This is called market information. Therefore, the game relative to payoff type direct mechanisms does not generate equilibrium allocations that could have been generated by the game in which any complex mechanisms are allowed. It is however difficult to incorporate agents' whole market information in the message space of mechanisms (Epstein and Peters (1999)).

[3]No payment upon no trading ($\mathbf{y}(0) = 0$) is implied by dual-additivity because condition 1 implies $\mathbf{y}(0) \leq 0$ and condition 2 implies $\mathbf{y}(0) \geq 0$.

[4]Note that a linear price schedule obviously induces non-decreasing average price because its average price is simply a constant. Terms of trade in the limit order markets for financial asset trading (Biais, Martimort, and Rochet (2000), Glosten (1994), Goettler, Parlour, and Rajan (2008), Hollifield, Miller, and Sandås (2004)) are characterized by a price schedule with non-decreasing average price. It is also observed in loan contracting when we interpret a price schedule as a schedule for interest rate.

share profits associated with trading with each agent.

We then show that any dual-additive price schedule (possibly a linear price schedule) can be supported as equilibrium terms of trade as long as it ensures that each firm receives no less profit than its reservation profit in the market where firms freely offer any arbitrary trading mechanisms that make quantity and monetary payment pairs contingent only on agents' messages. The key result of the paper is to show how to construct the equilibrium trading mechanisms for firms, given their implicit agreement on a dual-additive price schedule in a way that no firm gains by deviating to any arbitrary complex trading mechanism. This paper shows that each firm can offer a triggering weakly incentive compatible extended (WICE) direct mechanism to maintain their implicit agreement on a dual-additive price schedule, say $\tilde{\mathbf{y}}$. A triggering WICE direct mechanism asks each agent to report, along with the quantity that he wants to trade with the firm, whether there is a deviating firm and, if so, what would be the lowest average price that he believes he would face if he was the only one who traded with the deviating firm. An agent can correctly induce what would be the lowest average price given the terms of trade (i.e., price schedule) that he would face from the deviating firm if he was the only one who participated in the deviating firm. When agents are anonymous so that the trading mechanism is anonymous, each agent has the same belief on the lowest average price given the terms of trades (i.e., price schedule) that the deviating firm's trading mechanism would induce when he would be the only one who participated in the deviating firm's trading mechanism. As shown later, this approach is easily extended to the case in which agents are ex-ante heterogeneous.

The triggering WICE direct mechanism has the following structure. When two or more agents participate in a firm's triggering WICE direct mechanism, and more than half of their reports on the deviating firm's lowest average price are all p, then the firm offers a linear price schedule such that its unit price matches the minimum between p and the lowest average price of $\tilde{\mathbf{y}}$. In all other cases, the firm continues to offer $\tilde{\mathbf{y}}$.

Suppose that some firm indeed deviates to an arbitrary mechanism and each agent reports his true belief p to non-deviating firms. Then, each non-deviating firm's price schedule is the linear price schedule in which the unit price is equal to the minimum between p and the lowest average price of $\tilde{\mathbf{y}}$. When there are three or more agents, one agent cannot unilaterally change the non-deviating firm's price schedule given the other agents' truthful reports, p. When there are only two agents, one agent can unilaterally change the non-deviating firm's price schedule by reporting p' ($\neq p$) given the other agent's truthful report, p. In this case, the triggering WICE direct mechanism shoot them both by continuing to offer $\tilde{\mathbf{y}}$ to them. It not only makes each agent truthfully report p to each non-deviating firm given that the other agents do the same: It also makes it optimal for each agent to trade only with non-deviating firms. Consequently, a deviating firm ends up with its reservation profit upon any deviation to any arbitrary mechanism because no agents trade with the deviating firm in truthful continuation equilibrium.

When no firm deviates, each agent truthfully reports each firm, along with the quantity that he wants to trade with each firm, that no firm has deviated and then each firm continues to offer $\tilde{\mathbf{y}}$. Note that given the triggering WICE direct mechanisms, each firm offers the linear price schedule only when more than half of agents report that there is a deviating firm and its lowest average price given the terms of trade that they would face by trading with the deviating firm by himself. It implies that even when an agent unilaterally deviates to tell a lie, each firm continues to offer $\tilde{\mathbf{y}}$ so that no agent has an incentive to tell a lie. As long as a dual-additive price schedule ensure that each firm receives no less profit than its reservation profit, no firm has an incentive to deviate to any arbitrary trading mechanism because it only receives its reservation payoff upon deviation to any trading mechanism. Subsequently, multiple equilibrium prices associated with many different profit levels are possible in the competitive market even with the same probabilistic belief on the quality of a good.

Related Literature Biais, Martimort, and Rochet (2000) studied non-exclusive financial asset trading in a common agency framework. In their model, market makers compete in price schedules to supply liquidity to a single agent who is privately informed about the value of the asset and his hedging needs. With a continuum of the agent's types, they show that there exists a unique equilibrium in convex price schedules, which leads to Cournot-type equilibrium outcomes in the sense that each market maker makes positive expected profits but these profits go away as the number of market makers increases.

Prat and Rustichini (2003) extended non-exclusive trading to bilateral contracting with multiple firms and multiple agents in which a firm's terms of trade for an agent depends on the agent's message only but not the other agents' messages. However, agents have no private information in Prat and Rustichini's model. Recently Attar, Mariotti, and Salanié (2011) studied non-exclusive trading in the market for lemons in a common agency framework with multiple buyers and a single seller who is privately-informed about the quality of her good.[5] Then, they extended the results to bilateral contracting

[5] A Buyer is the contracting parties who offers a trading mechanism so he is equivalent

in which each buyer offers a menu of quantity and price pairs to each seller.[6] They provided a novel strategic foundation with bilateral contracting for the lemon's problem that Akerlof (1970) identified with ad hoc restrictions such as price-taking behavior and exclusive trading.[7] They showed that the lemon's problem is the necessary equilibrium feature in bilateral contracting in the sense that equilibrium aggregate allocation in bilateral contracting is unique, and the equilibrium price of the good is always equal to the expected quality of the good traded in the market, and a seller with a good of quality higher than the equilibrium price stays out of the market.[8]

Our paper studies equilibrium allocations in the fully generalized contracting environment where a firm's terms of trade for an agent can be determined based on communication with all agents. The results in our paper imply that in the general contracting environment, any allocation associated with dual-additive price schedule can be sustained as equilibrium allocation as long as it provides each firm with expected profit no less than its reservation profit. Therefore, the lemon's problem in the unique equilibrium allocation of bilateral contracting is no longer the necessary equilibrium feature in the general contracting environment. Combining the results in Attar, Mariotti, and Salanié (2011), it suggests that the lemon's problem can be the necessary equilibrium feature with contractual restrictions such as bilateral contracting but it arises as a coordination failure in the general contracting environment without contractual restrictions.

2 Model

Suppose that I ex-ante anonymous agents ($I \geq 2$) trade with J firms ($J \geq 2$) in a market for a good. Each agent can trade a good with any number of firms and each firm can also trade with any number of agents. Let x_i^j denote

to a firm in our paper. A seller is the contracting party who sends a report to buyers given trading mechanisms so she is equivalent to an agent in our paper.

[6]Han (2006) showed that menu theorems for common agency (Peters 2001, Martimort and Stole 2002) is extended to bilateral contracting.

[7]Rothchild and Stiglitz (1976) considered strategic contracting with multiple firms and a single agent but trading is exclusive in the sense that an agent trades with only one firm. They showed that the lemon's problem may be less severe in their screening model. However, equilibrium may not exist. Jaynes (1978) and Hellwig (1988) showed that when insurance firms directly disclose and share information on who accepts the insurance contract in non-exclusive contracting, the non-existence problem of equilibrium under exclusive contracting (Rothchild and Stiglitz 1976) can be resolved.

[8]Ales and Maziero (2009) has a similar result in the model with multiple firms and a single agent.

the quantity of the good that agent i trades with firm j. Let $X \subset \mathbb{R}$ be the set of feasible quantities that can be traded between each agent i and each firm j. Let m_i^j be the monetary payment from agent i to firm j. Let $(x_i, m_i) = (\sum_{k=1}^J x_i^k, \sum_{k=1}^J m_i^k) \in X \times \mathbb{R}$ be the pair of the total quantity that agent i trades with firms and the total monetary payment that he makes to firms. Let ω_i denote agent i's payoff type, which is assumed to be agent i's own private information. Let Ω be the set of all feasible payoff types for each agent. When agent i of type ω_i trades the total quantity x_i at the total payment m_i, his utility is $u(x_i, m_i, \omega_i)$. We assume that $u(x_i, m_i, \omega_i)$ is decreasing in m_i at each (x_i, ω_i). Each firm j's profit associated with $\mathbf{x}^j = [x_1^j, \ldots, x_I^j]$ and $\mathbf{m}^j = [m_1^j, \ldots, m_I^j]$ is denoted by $v^j(\mathbf{x}^j, \mathbf{m}^j, \omega)$ at each $\omega = [\omega_1, \ldots, \omega_I]$. Note that each firm j's profit function allows for the common value of each agent i's type.

Non-exclusive contracting is frequently observed in a variety of settings including investment financing, insurance, and trading goods or services:

Investment Financing: Entrepreneur i has a risky investment project. It generates profit $f(x_i)$ when the amount of money invested in the project is x_i. Let x_i^j be the amount of money borrowed from lender j and m_i^j the amount of money that the entrepreneur agrees to pay back when the project turns out to be successful. Let ω_i be the probability of success. Let ρ be the risk-free (gross) interest rate. Entrepreneur i's (expected) payoff is $u(x_i, m_i, \omega_i) = \omega_i[f(x_i) - m_i]$, where $(x_i, m_i) = (\sum_{k=1}^J x_i^k, \sum_{k=1}^J m_i^k)$. Lender j's (expected) profit associated with $\mathbf{x}^j = [x_1^j, \ldots, x_I^j]$ and $\mathbf{m}^j = [m_1^j, \ldots, m_I^j]$ is $v^j(\mathbf{x}^j, \mathbf{m}^j, \omega) = \sum_{k=1}^I \omega_k m_k^j - \rho \sum_{k=1}^I x_k^j$ at $\omega = [\omega_1, \ldots, \omega_I]$.

Insurance: Risk-averse individual i has total wealth W. Let $U(\cdot)$ be his Bernoulli utility function for money. An accident occurs with probability $1-\omega_i$. The accident entails a monetary loss L. Individual i pays insurance premium m_i^j to insurance company j and is reimbursed x_i^j in the case of the accident. The individual's expected utility is $\omega_i U(W - m_i) + (1-\omega_i)U(W - L - m_i + x_i)$, where $(x_i, m_i) = (\sum_{k=1}^J x_i^k, \sum_{k=1}^J m_i^k)$. The profit for insurance company j associated with $\mathbf{x}^j = [x_1^j, \ldots, x_I^j]$ and $\mathbf{m}^j = [m_1^j, \ldots, m_I^j]$ is $v^j(\mathbf{x}^j, \mathbf{m}^j, \omega) = \sum_{k=1}^I m_k^j - \sum_{k=1}^I (1-\omega_k) x_k^j$ at $\omega = [\omega_1, \ldots, \omega_I]$.

Trading: Each seller i produces a good. Let x_i^j be the quantity of the good sold to buyer j (firm) and $-m_i^j$ be the monetary payment made by buyer j. The quality of the good produced by seller i is his own private information and is denoted by ω_i. The cost of producing x_i units of the good to seller i is $c(x_i, \omega_i)$ so that seller i's payoff is $u(x_i, m_i, \omega_i) = -m_i - c(x_i, \omega_i)$. Buyer j's payoff associated with $\mathbf{x}^j = [x_1^j, \ldots, x_I^j]$ and $\mathbf{m}^j = [m_1^j, \ldots, m_I^j]$ is $v^j(\mathbf{x}^j, \mathbf{m}^j, \omega)$ at $\omega = [\omega_1, \ldots, \omega_I]$.

3 Terms of Trade for Non-Exclusive Trading

Terms of trade can be characterized by a price schedule in most trading observed in practice. Let a price schedule $\mathbf{y}\colon X \to \mathbb{R}$ characterize terms of trade in the market in the sense that the agent can trade with each firm according to it. It specifies the agent's payment to the firm as a function of the quantity that the agent trades with the firm. Let $\mathbf{Y}$ be the set of all feasible price schedules for the firm. We assume that any price schedule $\mathbf{y} \in \mathbf{Y}$ satisfies

$$\mathbf{y}(x) \geq 0 \text{ if } x > 0 \text{ and } \mathbf{y}(x) \leq 0 \text{ if } x < 0 \tag{1}$$

(1) implies that the average (unit) price $\frac{\mathbf{y}(x)}{x}$ at any non-zero quantity x is non-negative. This condition shows that if the agent buys the good from the firm, he pays a non-negative amount of money to the firm and that if the firm buys the good from the buyer, it pays a non-negative amount of money to the buyer. This condition is natural in that sense that a seller can refuse to sell if he is asked to pay a positive amount of money to a buyer for selling his product.

For the agent's non-exclusive trading with multiple firms without arbitrage arbitrage opportunity, we define the dual-additivity as follows.

Definition 1 *A price schedule* $\mathbf{y}$ *is* dual-additive *if,*

1. *for all* $x, x' \in \mathbb{R}_+$ *and all* $x, x' \in \mathbb{R}_-$*, it is super-additive, i.e.,*

$$\mathbf{y}(x + x') \geq \mathbf{y}(x) + \mathbf{y}(x'), \tag{2}$$

2. *and, for all* $x \in \mathbb{R}_+$ *and all* $x' \in \mathbb{R}_-$*, it is sub-additive, i.e.,*

$$\mathbf{y}(x + x') \leq \mathbf{y}(x) + \mathbf{y}(x'). \tag{3}$$

(2) implies $\mathbf{y}(x) \geq \mathbf{y}(x) + \mathbf{y}(0)$ with $x' = 0$ so that $\mathbf{y}(0) \leq 0$. (3) implies $\mathbf{y}(x) \leq \mathbf{y}(x) + \mathbf{y}(0)$ with $x' = 0$ so that $\mathbf{y}(0) \geq 0$. Therefore, a dual-additive price schedule $\mathbf{y}$ has $\mathbf{y}(0) = 0$, i.e., no payment upon no trading.

When terms of trade are characterized by a dual-additive price schedule in the market, it ensures that the agent can buy (sell) from (to) all firms without arbitrage opportunities. For simplicity, consider the case with two firms. Let

$\bar{x}$ be the total quantity that the agent wants to trade with firms. First of all, consider $\bar{x} = 0$. If the agent buys quantity $x > 0$ from one firm and sells x (i.e., $x' = -x$), part 2 of definition 1 implies that $0 = \mathbf{y}(0) \leq \mathbf{y}(x) + \mathbf{y}(x')$. Therefore, there is no arbitrage opportunity in the sense that it is not possible for the agent to make profits by buying the good from one firm and selling it to the other firm.

Consider $\bar{x} > 0$. Suppose that the agent buys quantity $x > 0$ from one firm and sells $-x'$ (i.e., $x' < 0$) to the other firm given $x + x' = \bar{x}$. Part 2 of Definition 1 implies that $\mathbf{y}(\bar{x}) \leq \mathbf{y}(x) + \mathbf{y}(x')$ so that it saves the agent's expenditure if the agent buys $\bar{x}$ from one firm instead of buying quantity x from one firm and selling $-x'$ to the other firm. Now suppose that the agent buys quantity $x > 0$ from one firm and quantity $x' > 0$ from the other firm given $x + x' = \bar{x}$, then part 1 of Definition 1 implies that $\mathbf{y}(\bar{x}) \geq \mathbf{y}(x) + \mathbf{y}(x')$ so that it saves expenditure if the agent buys x from one firm and x' from the other firm instead of buying $\bar{x}$ from one firm. It implies that if $\bar{x} > 0$, then it is expenditure saving to buy from every firm. One can also show the same argument above for $\bar{x} < 0$.

The argument above with two firms shows that there exists a solution to the agent's payoff maximization problem in which, without arbitrage opportunities, the agent buys (sells) the good from (to) every firm if he buys the good from one firm. This argument can be generalized to any number of firms. Assume that the market terms of trade are characterized by a price schedule $\mathbf{y}$ and hence the agent can trade with every firm according to $\mathbf{y}$. Then, the payoff maximization problem for each agent i of type ω_i is

$$\max_{(x^1,\ldots,x^J)\in X} u\left(\sum_{k=1}^{J} x^k, \sum_{k=1}^{J} \mathbf{y}(x^k), \omega_i\right). \tag{4}$$

Theorem 1 establishes the agent's non-exclusive optimal trading for any given number of firms, $J \geq 2$.

Theorem 1 *For any given number of firms, $J \geq 2$, suppose that a dual-additive price schedule $\mathbf{y}$ characterizes the market terms of trade. Then, there exists a solution to the payoff maximization problem (4) in which, without arbitrage opportunities, the agent buys (sells) the good from (to) every firm if he buys (sells) the good from (to) one firm.*

Proof. Denote by x^j the quantity traded with each firm j so that $x = \sum_{k=1}^{J} x^k$ is the total quantity that the agent trades with all firms. We examine the expenditure minimization problem for the agent when he wants to trade the total quantity x with firms.

First of all, consider the agent's expenditure minimization problem for $x = 0$. Suppose that the agent trades negative quantities with firms in a non-empty subset N_0 and non-negative quantities with firms in a non-empty subset M_0 with $N_0 \cup M_0$ being equal to the whole set of firms. Take out firm n ($\neq j$) in N_0 and firm m in M_0. Because of (3), we have

$$\mathbf{y}(x^n + x^m) \leq \mathbf{y}(x^n) + \mathbf{y}(x^m)$$

so that it is no more costly for the agent to trade x^{n+m} with firm n and zero quantity with firm m. If $x^{n+m} < 0$, then put firm n back into N_0. If $x^{n+m} \geq 0$, then put firm n into M_0. Put firm m into M_0. After putting firm n and firm m into the proper subset of firms, let N_1 (M_1) be the subset of firms with which the agent trades negative (non-negative) quantities. After a finite number T of repeating this procedure, we have N_T with only one firm, say firm j, in it and M_T with any other firms in it. Furthermore only one firm, say firm ℓ, in M_T has $x^\ell = -x^j > 0$ and any other firm in M_T has zero quantity traded with the agent. Note that it is no more costly for the agent to trade corresponding quantities with firms in N_t and M_t than to trade corresponding quantities with firms in N_{t-1} and M_{t-1} for all $t = 1, \ldots, T$. Because $x^\ell = -x^j$, we have $\mathbf{y}(0) \leq \mathbf{y}(x^\ell) + \mathbf{y}(x^j)$. Therefore, it is no more costly for the agent to trade zero quantity with every firm than to trade corresponding quantities with firms in N_T and M_T. It shows that it is expenditure minimizing for the agent not to trade with any firm for $x = 0$ and therefore there are no arbitrage opportunities in the sense that the agent cannot make profits by buying the good from some firms and selling it to some other firms.

Secondly, consider $x > 0$. Suppose that there is a non-empty subset of firms N with which the agent trades negative quantities. Then, there must be a non-empty subset of firms M with which the agent trades non-negative quantities such that $N \cup M$ is the whole set of firms. For any $n \in N$ and any $m \in M$, it is no more costly for the agent to trade quantity $x^n + x^m$ with only one of firm n or firm m because we have $\mathbf{y}(x^n + x^m) \leq \mathbf{y}(x^n) + \mathbf{y}(x^m)$ according to (3). It implies that we can focus on non-negative quantities for each firm j when total quantity x that the agent trades with firms is positive. Now suppose that there is a firm with which the agent does not trade. Then, pick a firm, say firm j, with which the agent trades a positive quantity $x^j > 0$. Given $x^j > 0$, choose a pair of x and x' such that $x > 0, x' > 0$, and $x + x' = x^j$. (2) yields $\mathbf{y}(x^j) \geq \mathbf{y}(x) + \mathbf{y}(x')$. It implies that it is no more costly to trade quantity x with firm j and quantity x' with the firm with which the agent did not trade at all. Therefore, a solution for the agent' expenditure minimization problem for any $x > 0$ can be derived by focusing on positive quantities ($\mathbb{R}_{++}$) for each firm for the agent's expenditure minimization problem.

Lastly, we can similarly prove that a solution for the agent' expenditure minimization problem for any $x < 0$ can be derived by focusing on negative quantities ($\mathbb{R}_{--}$) for each firm for the agent's expenditure minimization problem. It completes the proof. ■

Theorem 1 shows that non-exclusive trading occurs as the agent's optimal trading decision when a dual-additive price schedule characterizes terms of trade in the market. Furthermore, a dual-additive price schedule ensures no arbitrage opportunities.

We can find sufficient conditions for a dual-additive price schedule. For any given price schedule $\mathbf{y}$, define the average price function $AP_{\mathbf{y}}\colon X\backslash\{0\} \to \mathbb{R}_+$ as follows

$$AP_{\mathbf{y}}(x) = \frac{\mathbf{y}(x)}{x} \text{ for all } x \in X\backslash\{0\}.$$

Proposition 1 shows that non-decreasing average price associated with a price schedule $\mathbf{y}$ with $\mathbf{y}(0) = 0$ is a sufficient conditions for a dual-additive price schedule.

Proposition 1 *If a price schedule* $\mathbf{y}$ *with* $\mathbf{y}(0) = 0$ *induces a non-decreasing average price function, then it is dual-additive.*

Proof. For any $x, x' \in \mathbb{R}_+$, we should show that (2) is satisfied.

Case 1 ($x = x' = 0$): Because of $\mathbf{y}(0) = 0$, we have $\mathbf{y}(x + x') = \mathbf{y}(x) = \mathbf{y}(x') = 0$ and hence (2) is satisfied.

Case 2 ($x > 0$ and $x' = 0$): Because $\mathbf{y}(x') = 0$ and $\mathbf{y}(x + x') = \mathbf{y}(x)$, (2) is satisfied. we can similarly prove that (2) is satisfied when $x = 0$ and $x' > 0$

Case 3 ($x > 0$ and $x' > 0$): (2) is satisfied because

$$\begin{aligned}\mathbf{y}(x + x') &= x \times AP_{\mathbf{y}}(x + x') + x' \times AP_{\mathbf{y}}(x + x') \\ &\geq x \times AP_{\mathbf{y}}(x) + x' \times AP_{\mathbf{y}}(x') = \mathbf{y}(x) + \mathbf{y}(x'). \quad (5)\end{aligned}$$

The inequality in (5) holds because both x and x' are positive and hence $AP_{\mathbf{y}}(x + x') \geq AP_{\mathbf{y}}(x)$ and $AP_{\mathbf{y}}(x + x') \geq AP_{\mathbf{y}}(x')$.

Now examine any $x, x' \in \mathbb{R}_-$. Because the case with $x = x' = 0$ is covered in Case 1, we only need to consider the following two cases.

Case 4 ($x = 0$ and $x' < 0$): Because $\mathbf{y}(x) = 0$ and $\mathbf{y}(x + x') = \mathbf{y}(x')$, (2) holds. Similarly, (2) can be shown when $x < 0$ and $x' = 0$.

Case 5 ($x < 0$ and $x' < 0$): Because the average price function is non-decreasing and both x and x' are negative, we have that $x \times AP_{\mathbf{y}}(x+x') \geq x \times AP_{\mathbf{y}}(x)$ and $x' \times AP_{\mathbf{y}}(x+x') \geq x' \times AP_{\mathbf{y}}(x')$. Subsequently, (5) holds and hence (2) is satisfied.

Finally, examine any $x \in \mathbb{R}_+$ and all $x' \in \mathbb{R}_-$. The case with $x = x' = 0$ is covered in Case 1. The case with $x > 0$ and $x' = 0$ is covered in Case 2. The case with $x = 0$ and $x' < 0$ is covered in Case 4. Therefore, we only need to consider the case with $x > 0$ and $x' < 0$.

Case 6 ($x > 0$ and $x' < 0$ with $x + x' \neq 0$): Non-decreasing average price means that $x \times AP_{\mathbf{y}}(x + x') \leq x \times AP_{\mathbf{y}}(x)$ and $x' \times AP_{\mathbf{y}}(x + x') \leq x' \times AP_{\mathbf{y}}(x')$. Subsequently, we have

$$\begin{aligned}\mathbf{y}(x + x') &= x \times AP_{\mathbf{y}}(x + x') + x' \times AP_{\mathbf{y}}(x + x') \\ &\leq x \times AP_{\mathbf{y}}(x) + x' \times AP_{\mathbf{y}}(x') = \mathbf{y}(x) + \mathbf{y}(x'),\end{aligned}$$

which proves (3).

Case 7 ($x > 0$ and $x' < 0$ with $x + x' = 0$): Because $x' = -x$ and $AP(x) \geq AP(x') \geq 0$, we have

$$\begin{aligned}0 \leq x \times (AP_{\mathbf{y}}(x) - AP_{\mathbf{y}}(x')) &= \\ x \times AP_{\mathbf{y}}(x) + x' \times AP_{\mathbf{y}}(x') &= \mathbf{y}(x) + \mathbf{y}(x').\end{aligned}$$

which proves (3) given $0 = \mathbf{y}(0) = \mathbf{y}(x + x')$.

Cases 1-7 covers all possible cases for x and x' and hence it completes the proof. ■

Clearly, a linear price schedule with $\mathbf{y}(0) = 0$ is dual-additive because its average price is a constant. A linear price schedule captures equilibrium terms of trade in Ales and Maziero (2009) and Attar, Mariotti, and Salanié (2011). Terms of trade in the limit order markets for financial asset trading (Biais, Martimort, and Rochet (2000), Glosten (1994), Goettler, Parlour, and Rajan (2008), Hollifield, Miller, and Sandås (2004)) can be characterized by a price schedule with $\mathbf{y}(0) = 0$ and non-decreasing average price. In the limit order markets, a price schedule $\mathbf{y}$ can be traced by $\mathbf{y}(x) = \int_0^x y(z)dz$, where $y(z)$ is the marginal price at which the firm trades the zth unit. If the price schedule $\mathbf{y}$ induces non-decreasing average price, then $y(z)$ is non-decreasing in z so that the sequence of marginal prices $y(z)$ amounts to a sequence of limit orders. The property of non-decreasing average price leads to equivalence between a price schedule and a sequence of limit orders and it reflects the execution priority of the limit sell (buy) orders placed at lower (higher) prices. Furthermore, it does not generate any arbitrage opportunity because it is dual-additive.

Non-exclusive contracting is also a prominent feature in loan contracting (e.g. financing a risky project) or some of insurance contracting (e.g. credit default swap). Suppose that a lender faces that the effective average cost of financing a risky project is non-decreasing in the amount of money that he lends, then the market terms of trade may show that the interest rate is non-decreasing in the amount of money that the lender lends. The market terms of trade in non-exclusive insurance contracting may feature dual-additivity if the effective average cost of insurance is non-decreasing in the amount of repayment.

While a price schedule $\mathbf{y}$ with (i) $\mathbf{y}(0) = 0$ and (i) the property of the non-decreasing average price is dual-additive, dual-additivity is more general than the property of non-decreasing average price. Example 1 below shows that dual-additivity does not necessarily require the property of non-decreasing price.

Example 1 *Consider a price schedule* $\mathbf{y}$ *such that, for* $a \geq d > 0$

$$\mathbf{y}(x) = \begin{cases} 0 & \text{if } x < a \\ a & \text{if } a \leq x < a+d \\ x & \text{otherwise} \end{cases} .$$

This price schedule exhibits decreasing average price in $[a + a + d)$ *but it is dual-additive.*

Proof. Consider the case with $x, x' \in \mathbb{R}_+$. If $x \geq a + d$ and $x' \geq a + d$, then $\mathbf{y}(x) + \mathbf{y}(x') = x + x' = \mathbf{y}(x + x')$. If $a \leq x < a + d$ and $x' \geq a + d$, then $\mathbf{y}(x) + \mathbf{y}(x') = a + x' \leq x + x' = \mathbf{y}(x + x')$. If $0 \leq x < a$ and $x' \geq a + d$, then $\mathbf{y}(x) + \mathbf{y}(x') = 0 + x' \leq x + x' = \mathbf{y}(x + x')$. If $a \leq x < a + d$ and $a \leq x' < a + d$, then $\mathbf{y}(x) + \mathbf{y}(x') = a + a \leq x + x' = \mathbf{y}(x + x')$. If $0 \leq x < a$ and $a \leq x' < a + d$, then $\mathbf{y}(x) + \mathbf{y}(x') = 0 + a \leq \mathbf{y}(x + x')$. If $0 \leq x < a$ and $0 < x' < a$, then $\mathbf{y}(x) + \mathbf{y}(x') = 0 + 0 \leq \mathbf{y}(x + x')$. Hence (2) is satisfied when $x, x' \in \mathbb{R}_+$.

(2) is also satisfied when $x, x' \in \mathbb{R}_-$ because $\mathbf{y}(x) + \mathbf{y}(x') = 0 + 0 = \mathbf{y}(x + x')$ for all $x, x' \in \mathbb{R}_-$. Finally, consider the case with $x \in \mathbb{R}_+$ and $x' \in \mathbb{R}_-$. Note that

$$\mathbf{y}(x) + \mathbf{y}(x') = \mathbf{y}(x) \tag{6}$$

because $x' \leq 0$. We have

$$\mathbf{y}(x) \geq \mathbf{y}(x + x') \tag{7}$$

because $x \geq x + x'$ and the price schedule is a non-decreasing function. (6) and (7) yield $\mathbf{y}(x) + \mathbf{y}(x') \geq \mathbf{y}(x + x')$ so that (3) is satisfied when $x \in \mathbb{R}_+$ and $x' \in \mathbb{R}_-$. ■

Example 1 shows that the property of dual-additivity is weaker than the property of non-decreasing average price with $\mathbf{y}(0) = 0$. Based on Proposition 1, we can deduce the following result associated with a convex price schedule

Corollary 1 *Any convex price schedule* $\mathbf{y}$ *with* $\mathbf{y}(0) = 0$ *is dual-additive.*

Proof. The three-slope inequality (Fact 2.1.1 in Borwein and Vanderwerff (2010)) is stated as follows. Suppose that $f\colon \mathbb{R} \to (-\infty, +\infty]$ is convex and $x < y < z$. Then

$$\frac{f(y) - f(x)}{y - x} \leq \frac{f(z) - f(x)}{z - x} \leq \frac{f(z) - f(y)}{z - y}$$

whenever x, y and z are in the domain of f.

Suppose that $\mathbf{y}$ is convex with $\mathbf{y}(0) = 0$. If $z = 0$, the three-slope inequality implies that for $x < y < 0$,

$$\frac{\mathbf{y}(y) - \mathbf{y}(x)}{y - x} \leq \frac{\mathbf{y}(0) - \mathbf{y}(x)}{0 - x} \leq \frac{\mathbf{y}(0) - \mathbf{y}(y)}{0 - y}, \text{ so } \frac{\mathbf{y}(x)}{x} \leq \frac{\mathbf{y}(y)}{y}.$$

If $y = 0$, the three-slope inequality implies that for $x < 0 < z$,

$$\frac{\mathbf{y}(0) - \mathbf{y}(x)}{0 - x} \leq \frac{\mathbf{y}(z) - \mathbf{y}(x)}{z - x} \leq \frac{\mathbf{y}(z) - \mathbf{y}(0)}{z - 0}, \text{ so } \frac{\mathbf{y}(x)}{x} \leq \frac{\mathbf{y}(z)}{z}.$$

If $x = 0$, the three-slope inequality implies that for $0 < y < z$,

$$\frac{\mathbf{y}(y) - \mathbf{y}(0)}{y - 0} \leq \frac{\mathbf{y}(z) - \mathbf{y}(0)}{z - 0} \leq \frac{\mathbf{y}(z) - \mathbf{y}(y)}{z - y}, \text{ so } \frac{\mathbf{y}(y)}{y} \leq \frac{\mathbf{y}(z)}{z}.$$

Therefore, $\mathbf{y}$ induces a non-decreasing average price function. The dual-additivity of $\mathbf{y}$ then follows from Proposition 1. ■

Corollary 1 is proved by showing that any convex price schedule $\mathbf{y}$ with $\mathbf{y}(0) = 0$ induces non-decreasing average price. While convexity of a price schedule $\mathbf{y}$ with $\mathbf{y}(0) = 0$ is a sufficient condition for non-decreasing average price, non-decreasing average price does not require convexity of a price function with $\mathbf{y}(0) = 0$.

Example 2 *Consider a price schedule* $\mathbf{y}$ *such that*

$$\mathbf{y}(x) = \begin{cases} 0.9x & \text{if } x < 1 \\ x & \text{otherwise} \end{cases}.$$

It satisfies $\mathbf{y}(0) = 0$ *and induces a non-decreasing average function. However, it is not a convex function.*

Proof. It is easy to show that the price schedule $\mathbf{y}$ has a non-decreasing average price function. To show that it is not a convex function, we need to show that there exist x, x' and $\lambda \in [0, 1]$ such that

$$\mathbf{y}(\lambda x + (1 - \lambda)x') > \lambda \mathbf{y}(x) + (1 - \lambda)\mathbf{y}(x'). \tag{8}$$

(8) is satisfied at $(x, x', \lambda) = (0.9, 1, 1, 0.5)$ because $\mathbf{y}(\lambda x + (1 - \lambda)x') = 1$ and $\lambda \mathbf{y}(x) + (1 - \lambda)\mathbf{y}(x') = 0.955$. ■

Example 2 shows that the property of non-decreasing average price associated with a price schedule $\mathbf{y}$ with $\mathbf{y}(0) = 0$ is weaker than the convexity of a price schedule $\mathbf{y}$ with $\mathbf{y}(0) = 0$.[9]

4 Competition in Trading Mechanisms

We consider a market for non-exclusive trading in which firms may freely offer buyers any trading mechanisms that they want.[10] They do not observe trading mechanisms offered by competing firms. An alternative interpretation is that firms do observe competing firms' trading mechanisms but they cannot

[9] Biais et at. (2000) explained such equivalence between a price schedule and a sequence of limit orders when a price schedule is convex. In fact, we can show such equivalence based on the weaker property of a price schedule, i.e., non-decreasing average price.

[10] Yamashita (2010) characterizes the set of equilibrium payoffs that are supportable in competing mechanism games, using recommendation mechanisms. While it shows how agents can truthfully tell what type direct mechanism the principal should implement upon a principal's deviation, (i) the approach works only with three or more agents, (ii) a message becomes increasingly complicated in the number of agents because the agent should report a type direct mechanism, and (iii) the equilibrium payoff is not well identified because it is based on the principal's minmax value relative to complex mechanisms. The approach in our paper features convenience in a large class of applications. Each agent's message is simple that it includes only the deviating principal's lowest average price and the quantity that the agent wants to buy. It also works for any multiple number of agents including the case of two agents and the set of equilibrium payoffs is defined in terms of each firm's reservation profit which is independent of trading mechanisms.

write binding contracts directly contingent on competing firms' offers that they observe.[11] However, firms can make their terms of trade for an agent contingent on all agents' reports in their trading mechanism.

A firm's trading mechanism determines the quantity and payment pair for each agent contingent on all agents' messages. For each firm j, let C be the set of messages available for each agent i. Because agents are ex ante anonymous, the firm offers an anonymous trading mechanism. Given firm j's trading mechanism $\gamma^j : C^I \to X \times \mathbb{R}$, $\gamma^j(c_i^j, c_{-i}^j) \in X \times \mathbb{R}$ denotes the quantity and payment pair for each agent i when his message is c_i^j and the other agents' messages are c_{-i}^j. For notational simplicity, let C include the null message $\varnothing$. We assume that if an agent decides not to participate in firm j's trading mechanism, it is equivalent to sending the null message $\varnothing$ to firm j. Let $\gamma^j(C, c_{-i}^j)$ be the set of all quantity and monetary payment pairs that each agent i can induce by sending messages in C when the other agents' messages are c_{-i}^j.

Let Γ^j be the set of all feasible trading mechanisms for each firm j. Let $\Gamma \equiv \times_{k=1}^J \Gamma^k$. A competing mechanism game relative to Γ starts when each firm j simultaneously offers a trading mechanism from Γ^j. After observing a profile of trading mechanisms, each agent sends messages, one for each firm. Each firm j decides quantity and monetary payment pairs, one for each agent, contingent on the messages that it receives from agents. A trading mechanism can be very complex because the set of messages in a trading mechanism can be quite general in the degree and nature of the communication that it permits regarding what the other firms are doing: It could ask the agent to report not only about her type but also about the whole set of trading mechanisms offered by the other firms, the terms of trade that the agent chooses from the other firms, and so on.

Firms may want to offer complex trading mechanisms that provide agents with incentives to truthfully report what is happening with the other firms. By offering such sophisticated trading mechanisms, firms can actively punish a deviating firm by changing their terms of trade contingent on the agents' true reports on what the deviating firm is doing regardless of the complexity of the deviating firm's trading mechanism. This may give us the new insight into the set of equilibrium terms of trade that can be sustained when firms are able to offer sophisticated trading mechanisms. We are interested in robust perfect Bayesian equilibrium (henceforth simply equilibrium) allocations that

[11] Peters and Szentez (2011) is a notable exception. They consider the contracting environment where firms have full commitment: Firms make mechanisms publicly observable and they commit to make their contracts directly contingent on the competing firms' contracts.

persist even if any firm can deviate to any complex mechanism and equilibrium trading mechanisms that support robust equilibrium allocations.

Now we examine how firms can maintain their implicit collusion even when they cannot write binding contracts directly contingent on competing firms' offers that they observe. Suppose that the market terms of trade are characterized by a dual-additive price schedule $\tilde{\mathbf{y}}$. For the payoff maximization problem

$$\max_{(x^1,\dots,x^J)\in X} u\left(\sum_{k=1}^{J} x^k, \sum_{k=1}^{J} \tilde{\mathbf{y}}(x^k), \omega_i\right) \tag{9}$$

for each agent i of type ω_i based on the price schedule $\tilde{\mathbf{y}}$, let $(\tilde{x}^1(\omega_i),\dots,\tilde{x}^J(\omega_i))$ be a solution in which the agent buys (sells) from (to) all firms if he buys (sells) from (to) one firm. Let

$$\tilde{U}(\omega_i) \equiv u\left(\sum_{k=1}^{J} \tilde{x}^k(\omega_i), \sum_{k=1}^{J} \tilde{\mathbf{y}}(\tilde{x}^k(\omega_i)), \omega_i\right)$$

be the maximum payoff for agent i of type ω_i. Let $u_\circ(\omega_i) \equiv u(0,0,\omega_i)$ be the reservation payoff for the agent of type ω_i. Because $\tilde{\mathbf{y}}$ is dual-additive, $\tilde{\mathbf{y}}(x) = 0$ if $x = 0$ so that we can assure that $\tilde{U}(\omega_i) \geq u_\circ(\omega_i)$ for all ω_i.

Given a dual-additive price schedule for each agent, the expected payoff for firm j can be accordingly expressed as

$$V^j(\tilde{\mathbf{y}}) \equiv \mathbb{E}\left[v^j(\tilde{x}^j(\omega_1),\dots,\tilde{x}^j(\omega_I),\tilde{\mathbf{y}}(\tilde{x}^j(\omega_1)),\dots,\tilde{\mathbf{y}}(\tilde{x}^j(\omega_I)),\omega)\right],$$

where $\mathbb{E}[\cdot]$ is the expectation operator over $\omega = [\omega_1,\dots,\omega_I]$. Let $v^j_\circ \equiv \mathbb{E}[v^j(0,\dots,0,0,\dots,0,\omega)]$ be the reservation profit for firm j when it does not trade at all. The level of the firm's reservation profit depends on the application we consider. If the firm is a (potential) trader who owns a good such as a car owner or an asset holder, then its reservation profit is its payoff associated with keeping the good. If the firm is a producer that can make a production decision contingent on contracting with buyers, then its reservation profit is simply the zero profit associated with producing nothing.

We now examine how firms can implicitly support any dual-additive price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j, as their equilibrium terms of trade. To this end, we first construct each firm's equilibrium trading mechanism that prevents any firm's deviation to any complex trading mechanism. We call it a triggering WICE direct mechanism.

For an arbitrary dual-additive price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j, each firm j's triggering WICE direct mechanism is denoted by $\gamma^j_E\colon E^I \rightarrow$

$X \times \mathbb{R}$. The set of messages available for each agent i is $E \equiv P \times X$, where $P = \mathbb{R}_+ \cup \{\eta\}$. Each agent i reports $(p, x) \in E$.[12] The message $x \in X$ is the quantity that the agent wants to trade with the firm. The message p has the following meaning. If $p = \eta$, then it means either (i) no other firms deviate from the triggering WICE direct mechanisms or (ii) a deviating firm's price schedule for each agent is $\tilde{\mathbf{y}}$ and it is independent of the other agent's messages to the deviating firm. If $p \in \mathbb{R}_+$, then it means (a) there exists a deviating firm whose trading mechanism does not induce (ii) and (b) p is the the deviating firms' lowest average price for the agent if he was the only agent who participated in the deviating firm's mechanism.

Suppose that firm k deviates to a mechanism $\gamma^k \colon C^I \to X \times \mathbb{R}$. When each agent i is the only agent who participates in the deviating firm's mechanism, the deviating principal's lowest average price for the agent is defined as

$$\inf \left\{ p' \in \mathbb{R}_+ \colon p' = \frac{m}{x} \text{ for } (x, m) \in \gamma^k(C, \varnothing^{I-1}) \text{ and } x \neq 0 \right\}.$$

For an arbitrary dual-additive price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j, the triggering WICE direct mechanism $\tilde{\gamma}^j_E \colon E^I \to X \times \mathbb{R}$ has the following properties:

D1. If the number of participating agents is two or more and more than half of participating agents report $p \in \mathbb{R}_+$ for some other firm's lowest average price, the firm offers a linear price schedule $\tau(p)$ such that $\tau(p)(x) = ax$ for all $x \in X$. Each participating agent i then pays $\tau(\mathbf{y})(x) = ax$ for the quantity x that he submits along with his report on some other firm's lowest average price.

D2. In all other cases, γ^j_E ignores agents' reports on some other firm's price schedule and the price schedule continues to be $\tilde{\mathbf{y}}$. Each agent i then pays $\tilde{\mathbf{y}}(x)$ for the quantity x that he submits along with his report on some other firm's price schedule.

The key to the triggering WICE direct mechanism is to set up $\tau(p)$ for all $p \in \mathbb{R}_+$ in a way that it induces agents not to trade with a deviating firm in truth-telling continuation equilibrium. As shown later, non-deviating firms' triggering WICE direct mechanisms in fact lead to truth-telling continuation

[12] If the agent decides not to participate in a triggering WICE direct mechanism, then it is assumed to be equivalent to sending $x = 0$. Accordingly the mechanism assigns zero monetary payment for the agent.

equilibrium in which each agent reports to each non-deviating firm the lowest average price p that he believes he would face from the deviating firm if he was the only one who participated in the deviating firm's trading mechanism.

Suppose that a deviating firm's price schedule is $p \in \mathbb{R}$ for each agent if he was the only one who traded with the deviating firm. When two or more agents participate in the non-deviating firm's triggering WICE direct mechanism and more than half of participating agents report $p \in \mathbb{R}_+$, then the triggering WICE direct mechanism assigns the linear price schedule $\tau(p)(x) = ax$ that satisfies

$$a = \min\left[p, \inf_{x \in X \setminus \{0\}} \left(\frac{\tilde{\mathbf{y}}(x)}{x}\right)\right]. \tag{10}$$

Note that $\inf_{x \in X \setminus \{0\}} \left(\frac{\tilde{\mathbf{y}}(x)}{x}\right)$ is the lowest average price based on the price schedule $\tilde{\mathbf{y}}$.

Consider an arbitrary dual-additive $\tilde{\mathbf{y}}$ that induces $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j. Our main result shows that when every firm offers the triggering WICE direct mechanism with $\tau(\cdot)$ that satisfies (10) for any $p \in \mathbb{R}_+$, there exists the truth-telling continuation equilibrium in which no firm j can make more profit than $V^j(\tilde{\mathbf{y}})$ by deviating to any complex trading mechanism. Therefore, any dual-additive price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j can be supported as equilibrium terms of trade in the market as long as it generates non-negative expected profits for firms.

Theorem 2 *Suppose that each firm offers the triggering WICE direct mechanism associated with a dual-additive price schedule $\tilde{\mathbf{y}}$ with $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for all j. It is the equilibrium mechanism for each firm in perfect Bayesian equilibrium in which the truth-telling continuation equilibrium is characterized as follows:*

1. *When no firm deviates or firm k deviates to a mechanism that induces $\tilde{\mathbf{y}}$ to each agent regardless of the other agents' reports to firm k, each agent i of type ω_i sends the message $(\eta, \tilde{x}^j(\omega_i))$ to each non-deviating firm j and a message, to firm k, which leads him to trade $\tilde{x}^k(\omega_i)$ at $\tilde{\mathbf{y}}(\tilde{x}^k(\omega_i))$ with firm k.*

2. *When firm k deviates to any other mechanism, each agent i of type ω_i trades $\hat{x}(\omega_i)$ only with every non-deviating firm by reporting $(p, \hat{x}(\omega_i))$, where p is each agent's belief on the lowest average price that the deviating firm's mechanism would induce if only one agent participated in its mechanism and $\hat{x}(\omega_i)$ satisfies*

$$\hat{x}(\omega_i) \in \arg\max_x u((J-1)x, (J-1)\tau(p)(x), \omega_i).$$

Proof. Choose an arbitrary dual-additive price schedule $\tilde{\mathbf{y}}$ that induces $V^j(\tilde{\mathbf{y}}) \geq v^j_\circ$ for each firm j based on the solution $(\tilde{x}^1(\omega_i), \ldots, \tilde{x}^j(\omega_i))$ to problem (9). Each firm offers the triggering WICE direct mechanism associated with the dual-additive price schedule $\tilde{\mathbf{y}}$. We will show that the triggering WICE direct mechanism is the equilibrium trading mechanism for each firm in perfect Bayesian equilibrium in which agents truthfully communicate with non-deviating firms on their beliefs on the lowest average price that a deviating firm's trading mechanism would induce no matter how complex the deviating firm's trading mechanism is. First of all, consider truth-telling continuation equilibrium on the equilibrium path

(a) On the equilibrium path: When no firm deviates from its triggering WICE direct mechanism, assume that each agent i participates in all firms' triggering WICE direct mechanisms by sending the message $(\eta, \tilde{x}^j(\omega_i))$ to each firm j. Suppose that an agent considers a deviation from the report η when he communicates with a firm. Because of condition (D2), an agent cannot unilaterally change a firm's price schedule away from $\tilde{\mathbf{y}}$ with any other report in P given that the other agents all send η to the firm. Therefore, it is incentive compatible for each agent to send η to each firm when the other agents also send η to each firm. Because each firm's price schedule becomes $\tilde{\mathbf{y}}$, it is in fact optimal for each buyer i of type ω_i to participate in each firm j's triggering WICE direct mechanisms by sending $\tilde{x}^j(\omega_i)$ along with η.

(b) Off the equilibrium path: Now we consider firm k's deviation to any complex trading mechanism. There are two types of deviation.

(b-1) Suppose that firm k deviates to a trading mechanism $\gamma^k\colon C^I \to X \times \mathbb{R}$ such that (i) for all $c^k_i \in C$ and all $c^k_{-i}, \acute{c}^k_{-i} \in C^{I-1}$,

$$\gamma^k(c^k_i, c^k_{-i}) = \gamma^k(c^k_i, \acute{c}^k_{-i}) \tag{11}$$

and (ii) for each $x \in X$,

$$\min\{m \in \mathbb{R}\colon (x, m) \in \gamma^k(C, c^k_{-i})\} = \tilde{\mathbf{y}}(x). \tag{12}$$

(11) implies that the quantity and payment pair for each agent i depends only on his message but not on the other agents' messages. For any $c^k_{-i} \in C^{I-1}$, recall that $\gamma^k(C, c^k_{-i})$ denotes the set of all quantity and payment pairs that each agent i can induce from firm k.

When agent i chooses to trade x with firm k, there may be many messages that can induce the same quantity x along with different amounts of payment. If agent i ever chooses to trade x with firm k, it is always optimal for him to trade x at the minimum payment. Therefore, the left-hand side of (12) is the minimum payment that the agent can pay if he trades x with firm k. Note that (12) already presumes that firm k deviates to a mechanism in which the minimum on the left-hand side of (12) exists. In fact, when firm k deviates to a mechanism satisfying (11) and (12), it is equivalent to offering the price schedule $\tilde{\mathbf{y}}$.

Assume that given firm k's deviation to a mechanism satisfying (11) and (12), each agent i trades with all firms including the deviating firm. Each agent i of type ω_i sends the message $(\eta, \tilde{x}^j(\omega_i))$ to each non-deviating firm j and sends a message to firm k in a way that it induces him to trade $\tilde{x}^k(\omega_i)$ with the deviating firm at $\tilde{\mathbf{y}}(\tilde{x}^k(\omega_i))$. As proved in part (a), each agent finds it optimal to send η to each non-deviating firm when all the other agents send the message η to each non-deviating firm. This leads each non-deviating firm to assign the price schedule $\tilde{\mathbf{y}}$ given its triggering WICE direct mechanism. Because all firms' price schedules, including the deviating firm's, are $\tilde{\mathbf{y}}$, it is again optimal for each agent i of type ω_i to trade $\tilde{x}^\ell(\omega_i)$ with firm ℓ at $\tilde{\mathbf{y}}(\tilde{x}^\ell(\omega_i))$ for all $\ell = 1, \ldots, J$. Parts (a) and (b-1) complete the proof of the first part of Theorem 1.

(b-2) Suppose that firm k deviates to any other trading mechanism $\gamma^k \colon C^I \rightarrow X \times \mathbb{R}$ that does not belong to (b-1). Assume that agent i is the only one agent who participates in firm k's trading mechanism. Then, $\gamma^k(C, \varnothing^{I-1})$ is the set of all quantity and payment pairs that agent i can choose from firm k and hence the lowest average price for the agent becomes

$$p = \inf \left\{ p' \in \mathbb{R}_+ \colon p' = \frac{m}{x} \text{ for } (x, m) \in \gamma^k(C, \varnothing^{I-1}) \text{ and } x \neq 0 \right\}. \qquad (13)$$

We will show that upon firm k's deviation to a trading mechanism $\gamma^k : C^I \rightarrow X \times \mathbb{R}$, each agent i of type ω_i trades with only non-deviating firms by sending the message $(p, \hat{x}(\omega_i))$ to each non-deviating firm, where p satisfies (13) and $\hat{x}(\omega_i) \in \arg\max_x u((J-1)x, (J-1)\tau(\mathbf{y})(x), \omega_i)$.

When every agent reports p to each non-deviating firm, the non-deviating firm's price schedule becomes $\tau(p)$ according to (D1) so that the agent pays $\tau(p)(x) = ax$ for any x that the agent trades with the non-deviating firm. We first show that it is optimal for each agent to truthfully report p defined in (13) to each non-deviating firm when the other agents do the same.

Assume that all agents truthfully report p defined in (13) to each non-deviating firm upon firm k's deviation to $\gamma^k \colon C^I \rightarrow X \times \mathbb{R}$. Suppose that an agent reports p'' other than p to any non-deviating firm given that all other agents report p. If $I \geq 3$, then the non-deviating firm's price schedule is still $\tau(p)$ according to (D1) because still more than half of participating agents report p. Therefore, the agent has no incentive to deviate away from p. If $I = 2$, then the non-deviating firm's price schedule becomes $\tilde{\mathbf{y}}$ according to

(D2) because one agent reports p and the other agent reports p''. Subsequently, the agent pays $\tilde{\mathbf{y}}(x)$ for any x that the agent trades with the non-deviating firm. Because of (10), $\tau(p)$ satisfies $\tau(p)(x) = ax \leq \tilde{\mathbf{y}}(x)$ for any x. Hence even when $I = 2$, it is optimal for an agent to truthfully report p to each non-deviating firm given that the other agent does the same.

Finally we will show that it is optimal for each agent to trade $\hat{x}(\omega_i)$ only with each non-deviating firm. Suppose that agent i currently trades x with a non-deviating firm given that all agents report p to the non-deviating firm and that he is the only agent who trades with the deviating firm. Let x' be the quantity that agent i trades with the deviating firm. Then, the total payment associated with trading x with the non-deviating seller and x' with the deviating seller is no less than $ax + px'$ because of the definition of $\tau(p)$ in (10) and the definition of p in (13). However, if the agent trades $x + x'$ only with the non-deviating seller, the monetary payment is $a(x + x')$, which is no more than $ax + px'$ because of (10). It implies that the agent can trade $x + x'$ with no more monetary payment when he trades only with the non-deviating firm. Therefore, it is optimal for each agent not to trade with the deviating firm when all the other agents do not trade with the deviating firm. Because each non-deviating firm's price schedule is the linear price schedule, $\tau(p)$, it does not matter whether an agent trades with only one non-deviating firm or with every non-deviating firm. Subsequently, each agent i of type ω_i optimally participates in every non-deviating firms' triggering WICE direct mechanisms by sending $(p, \hat{x}(\omega_i))$. This completes the proof of the second part of Theorem 1.

When firm k deviates to a trading mechanism that belongs to (b-1), it receives the same expected profit $V^k(\tilde{\mathbf{y}})$ that it would receive with the triggering WICE direct mechanism. When firm k deviates to any other mechanism, i.e., one that belongs to (b-2), it receives its reservation profit $v^j_\circ$ because no agents trade with firm k in truthful continuation equilibrium. Because the expected profit $V^k(\tilde{\mathbf{y}})$ associated with the triggering WICE direct mechanism is no less than $v^j_\circ$, firm k cannot gain by deviating to any alternative mechanism. ■

When all firms maintain their triggering WICE direct mechanisms, their price schedules are $\tilde{\mathbf{y}}$ in truth-telling continuation equilibrium. When a firm deviates to an arbitrary mechanism that is essentially equivalent to offering $\tilde{\mathbf{y}}$ to each agent independent of the other agents' messages, non-deviating firms do not punish the deviating firm and their price schedules continue to be $\tilde{\mathbf{y}}$ in truth-telling continuation equilibrium. If a firm deviates to any other mechanism, then each agent reports the lowest average price p that the deviating firm's mechanism could induce if he participated in the deviating firm's trading mechanism alone in truth-telling continuation equilibrium. By offering a linear price schedule that has the unit price no higher than the minimum between the average unit prices of $\tilde{\mathbf{y}}$ and $\mathbf{y}$ upon agents' true reports, non-deviating firms can induce agents not to participate in the deviating firm's trading mechanism. Therefore, no firm j can find a profitable deviation to any trading mechanism as long as the firm's expected profit $V^j(\tilde{\mathbf{y}})$ associated with a dual-additive price schedule $\tilde{\mathbf{y}}$ is no less than $v^j_\circ$.

When there are three or more agents, it is straightforward for a non-deviating firm to detect an agent's unilateral deviation given that all the other agents report the true lowest average price that would be induced by a deviating firm's mechanism. When there are only two agents, the WICE triggering mechanism shoots both agents upon their different reports by continuing to assign $\tilde{\mathbf{y}}$ for both agents. In this way, the WICE triggering mechanism can induce truth-telling continuation equilibrium as long as there are multiple agents.[13]

Theorem 1 can be easily extended to ex-ante heterogeneous agents. Suppose that firms agree to offer an array of dual-additive price schedules $[\tilde{\mathbf{y}}_1, \ldots, \tilde{\mathbf{y}}_I]$ for agents, where $\mathbf{y}_i$ is for agent i. Let $u_i(\cdot, \cdot, \omega_i)$ is the payoff function for agent i of type ω_i. For the payoff maximization problem

$$\max_{(x_i^1,\ldots,x_i^J)\in X} u_i\left(\sum_{k=1}^{J} x_i^k, \sum_{k=1}^{J} \tilde{\mathbf{y}}_i(x_i^k), \omega_i\right)$$

for each agent i of type ω_i based on the dual-additive price schedule $\tilde{\mathbf{y}}_i$, we can find a solution $(\tilde{x}_i^1(\omega_i), \ldots, \tilde{x}_i^J(\omega_i))$ in which the agent buys (sells) from (to) all firms if he buys (sells) from one firm. Given an array of dual-additive price schedules $[\tilde{\mathbf{y}}_1, \ldots, \tilde{\mathbf{y}}_I]$, one can construct the triggering WICE direct mechanism for each firm j that asks agents to report an array of the lowest average prices $[p_1, \ldots, p_I]$, along with their quantities, such that p_i is the lowest average price that agent i would face if he was the only one who participated in a deviating firm's trading mechanism.

Consider the case in which the number of participating agents is two or more, and more than half of their reports on some other firm's lowest average prices, one for each agent i, are all $[p_1, \ldots, p_I]$ and $p_i \neq \eta$ for some i. The

[13] As in the single principal case, an equilibrium in competing mechanism games is derived by the truth-telling continuation equilibrium in which agents truthfully reports on what principals ask. The notion of the strongly robust equilibrium was studied in Han (2007) and Peters (2001). An equilibrium is said to be strongly robust if it survives in all continuation equilibria upon any firm's deviation. Attar, Mariotti, and Salanié (2011) pointed out that strongly robustness is too demanding and especially it is inconsistent with equilibrium in the market for lemons (i.e., the common-value environment).

triggering WICE direct mechanism assigns the price schedule $\tau_i(p_i)$ for agent i is linear and it satisfies (i) $\tau_i(p_i)(x) = a_i x$ and

$$a_i = \min\left[p_i, \inf_{x \in X \setminus \{0\}} \left(\frac{\tilde{\mathbf{y}}_i(x)}{x}\right)\right].$$

In all other cases, the triggering WICE direct mechanism continues to assign $\tilde{\mathbf{y}}_i$ for each agent i. Given this triggering WICE direct mechanism, we can show that Theorem 1 is extended for ex-ante heterogeneous agents.

5 Conclusion

Akerlof showed the multiplicity of equilibria in the competitive markets with adverse selection in his breakthrough paper in 1970. When the quality of the good is not observable, the Walrasian market price reflects only the average quality of the good so that we may have multiple fixed-point Walrasian prices that lead to different average quality of the good traded in the market. Because the Walrasian market price correctly reflects the average quality of the good, equilibrium profits are zero in any equilibrium.

Our results differ from those in Akerlof. First of all, we showed that firms can maintain a wide range of collusive outcomes through the sophisticated trading mechanisms that make firms' terms of trade responsive to agents' report on competing firms' lowest average price. Subsequently, various levels of positive equilibrium profits and corresponding prices may arise given the same probabilistic beliefs on the quality of a good regardless of the number of firms in the market. Secondly, our multiplicity of equilibrium prices and profits is based on the full game-theoretical approach in which firms can freely offer any trading mechanisms. Not only does it allow firms to use sophisticated trading mechanisms as their equilibrium mechanisms but it also allow firms to deviate to any complex trading mechanisms if they want. Multiple equilibria in our paper do not disappear even when firms can deviate to any complex trading mechanism.

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