THE SAVING BEHAVIOUR OF A TWO PERSON HOUSEHOLD

Martin Browning
McMaster University

First draft: May 1994

I thank John Burbidge for discussion and the SSHRC for support.

ABSTRACT

Wives are typically younger than their husbands and women typically live longer than men. These two facts mean that for a typical married couple, wives have more incentive to save for old age than do husbands. This paper presents a theoretical model of the determination of household saving and portfolio choice taking into account differences in preferences for saving. The model is a non-cooperative game in which each person can use their own current consumption to contribute to current (household) consumption or to a range of assets.

The results derived are in marked contrast to 'unitary' models of intertemporal allocation that assume a single household utility function and conclude that saving is unaffected by the distribution of income within the household. The most important result is that the level and the composition (portfolio) of saving and the time path of consumption is highly dependent on the distribution of income within the household. It is also shown that the introduction of an actuarially fair state pension scheme may have non-neutral effects on saving.

1. INTRODUCTION

Women typically live longer than men. Wives are typically younger than their husbands. These two facts imply that preferences about saving for the future in 'married' households with forward looking members will differ between husband and wife. This paper presents an analysis of saving and portfolio behaviour if the members of the household have different preferences over future consumption as compared to current consumption. The analysis presented below stands in contrast to 'unitary' models of household intertemporal allocation which assume that the household has a single utility function. Thus this paper is a contribution in a novel context to the burgeoning literature on intra-household decision making.

The base case analysis is for a simple two person (husband and wife¹), two period model in which there is only one asset - saving. It is assumed that because of the two facts given at the start of the paper the husband discounts the future more than the wife. In section 2 the implications of a non-cooperative decision making process for the base case are given². In this process each person chooses a contribution to first period (public) consumption from her or his private income; what is left over is saved for second period (public) consumption. We show that the household marginal propensity to consume depends on the distribution of income within the household and who receives any extra income. These two predictions are in opposition to the unitary model. We also show that our non-cooperative process leads to efficient outcomes and that any efficient outcome can be supported by our non-cooperative process for some distribution of within household income. Thus there is no conflict with models of intra-household allocation that impose efficiency (so-called 'collective' models - see Chiappori (1988, 1992) and Browning and Chiappori (1994)).

In section 3 we present some data to illustrate how strong the effects discussed might be. We show that for the typical married couple the wife can

¹ The terminology is for convenience; the analysis obviously applies in a wider context than just married couples.

² See Ulph (1988) for a non-cooperative model of demand.

expect to live 50% longer in the retirement period (defined as the husband being 65 or over) if both survive to the retirement date. For plausible preferences over intertemporal allocation this means that wives will prefer a household saving rate that is considerably higher than that of the husband. If this is the case then the distribution of income within the household can have a strong effect on household savings behaviour. Some data on the within household distribution of income is also presented to give some idea of how important all of this might be.

In section 4 a new asset is introduced - an actuarially fair state pension. It is shown that this can have an effect on the time path of household consumption. Once again this is in marked contrast to unitary models which predict that households will simply undo the effects of an actuarially fair state pension scheme and achieve the same path of consumption as with no pension.

In section 5 the state pension is dropped but we introduce two new saving instruments - actuarially fair annuities and insurance. Although annuities are fairly rare, employer provided pensions are not. If the latter have poor survivor benefits (as most such pension schemes seem to have) then they act just like annuities. We find that the household portfolio allocation between private saving, annuity and insurance depends very strongly on the distribution of income within the household and that the effect of an extra dollar of income on portfolio choice depends a great deal on who receives the extra income. Once again this is all in marked contrast to the unitary model.

In the concluding section some of the implications of all of this for empirical work are drawn out. This is clearly the next step - it is one thing to suggest that something is theoretically possible and quite another to show that it is empirically important. Any empirical work, however, has stringent data requirements. We show that the major effects of differential incomes are on portfolio choice rather than on consumption paths so that we will need data that has good information on the availability and use of annuities (or employer provided pension schemes) as well as good asset or consumption data.

We shall also need accurate information on the extent of survivor benefits for any employer provided pension scheme.

2. THE BASIC MODEL

We employ a very stylised two person (H and W for husband and wife respectively) model. A discussion of the assumptions follows immediately after the presentation of the basic model. We assume that there are two periods (1 and 2) and that the wife survives to the end of period 2 with certainty. The husband, however, faces a probability of survival equal to λ at the end of period one. We assume that $0 < \lambda < 1$. This is the only source of uncertainty in the model. The utility functions of the two people are:

$$u_H = v(C_1) + \lambda v(C_2)$$

$$u_W = \phi(C_1) + \phi(C_2)$$

where C_t is the consumption of a public good in period t . Note that the only discount factor is for H and this simply takes account of the probability of survival. We make the following assumptions on preferences.

Assumption 1. $v(\cdot)$ and $\phi(\cdot)$ are strictly increasing, strictly concave and twice continuously differentiable with $\phi(C) \rightarrow -\infty$ and $v(C) \rightarrow -\infty$ as $C \rightarrow 0$ from above.

In the context of an intertemporal model these are widely used restrictions on preferences.

In period 1, H and W have exogenously given incomes Y_H and Y_W respectively. It is assumed that this is the only source of household income $Y = (Y_H + Y_W)$. We denote the wife's share in household income by ρ , so that

$$Y_H = (1-\rho)Y \quad \text{and} \quad Y_W = \rho Y \quad \text{where } 0 \leq \rho \leq 1.$$

Period 2 household income is simply what is saved from period 1. In the following we sometimes refer to periods 1 and 2 as 'pre-retirement' and 'retirement' respectively. The real rate of interest is set to zero so that the budget constraint is:

$$C_2 = Y - C_1.$$

Before going on to the manipulation of this simplest model I discuss the assumptions made.

The model given above is the one that we shall be using in most of the subsequent analysis. It turns out that the assumptions made can be relaxed in many realistic directions without changing the (qualitative) conclusions of the model. The structure above has been chosen to minimise on notation. The assumptions made are:

(i) neither person cares about the other. We could relax this by allowing each person to have a 'household welfare function' that takes weighted sums of the two utility functions as the representations of their preferences. So long as the weight each person attaches to the other is less than the weight they attach to themselves, nothing qualitative changes in the subsequent analysis. In quantitative terms, the assumption of caring simply acts in the same way as an increase in the discount factor λ .

The obvious alternative assumptions to make to the one given here is that either there is perfect sympathy (both partners have the same utility function) or that one person acts as a dictator. Neither of these assumptions is consistent with the widespread finding in the demand literature that demand patterns depend strongly on who receives the income in the household (see Browning and Chiappori (1994) for references). The only way to reconcile these would be to assume that households have different decision processes for saving than they do for within period allocation. Although this has some superficial plausibility it is difficult to rationalise it within a neo-classical framework.

(ii) there is no discounting of the future except for H taking into account the probability of death. Once again, we could introduce an extra parameter for W's discount factor without changing anything below. The important point is that H discounts the future more. For the reasons discussed in the introduction this seems plausible. One important caveat concerns the case where some income is left to heirs (other than the wife if the husband

dies). If there is a bequest motive and this is stronger for husbands/fathers than for wives/mothers then this could reverse the weight attached to saving. Note, however, that what evidence there is on mothers' and fathers' concern for their children suggests that the bias goes the other way which reinforces the effects assumed here (see, for example, Thomas (1990)).

(iii) all consumption is assumed public. This is obviously unrealistic unless two really can live as cheaply as one. One simple (and appealing) way to relax this assumption is to assume that two people need expenditure $(1+\theta)C$ to be as well off materially as one person spending C (where $0 \leq \theta \leq 1$). That is, the equivalence scale for one extra person is θ . The extreme cases are: all consumption is private ($\theta = 1$) and all consumption is public ($\theta = 0$). Once again it turns out that nothing qualitative is changed in the following if we adopt this framework. Below we shall illustrate the differences that arise from adopting the more realistic assumption.

Although scaling consumption by equivalent adults is simple it may not capture all of the differences in going from a one person to a two person household. However, the adoption of the more sophisticated approach of allowing that the mix of public and private expenditures can vary leads to considerable complications since each person now has more decision variables. This will not be pursued here.

(iv) there are only two periods and W survives through both with certainty. I have not worked through the many period model but I would conjecture that nothing qualitative changes if we replace these assumptions by one that gives a higher probability of death for H at the end of each period. As will be seen below, the important feature is that in any time period, H discounts the future more.

(v) there is only one savings instrument with a zero real rate. Once again we could allow for a positive real rate with no gain in generality but a loss of parsimony of notation. Of course, if there are tax advantages to saving now and spending in retirement then the real rate should reflect these. The restriction to one instrument is most important. In the later sections we

allow for pensions, annuities and life insurance.

One important asset that I do not consider is the purchase of a house. Empirically this is one of the most important forms of saving for old age for many people. The incorporation of an extra good that is both a source of consumption flows and an asset is beyond the scope of the present paper.

(vi) the period one incomes are given exogenously. As we shall see below, individual income is valued not just because it contributes to household income (and hence to consumption) but also because it gives power over the savings decision. Hence both partners have incentives that are stronger than the usual ones to increase their income. I also abstract from the retirement date choice. I regard the incorporation of endogenous incomes and retirement dates to be amongst the most important future extensions for this model but for now I stick with the assumption made.

(vii) all first period income is attributable to husband or wife. Effectively this rules out any starting wealth which would be considered as joint income. The reason I have excluded this is because I cannot think of a plausible way to assign the rights to spend this. However, the results below also hold for a model with wealth in which it is assumed that each person's share of wealth is monotone in their share of income.

(viii) if the husband survives then the household stays together. If there is some possibility that the husband and wife do not stay together then preferences over first period consumption will depend on the division of any saving between the two partners. Although this should be an important element in models of non-unitary households, we do not pursue it here. We note once again, however, that if men discount the future more then the conclusions given below are likely to be robust to including the possibility of dissolution of the marriage.

It will be clear that in the framework given above husband and wife may have different views as to how much of household income to save for the future. In the extreme case where $\lambda = 0$, the husband will not want to save

anything which will be disastrous for the wife. Thus we need to posit how the household comes to make decisions. Here I adopt a Nash approach: each person J contributes to consumption in period 1 from Y_J ; any income left over is saved. As we shall see below, this non-cooperative approach not only leads to efficient outcomes but it can also be seen as a way of supporting any efficient outcome.

We have the following problems for the two agents:

$$(PH) \max_P v(Y-P-S) + \lambda v(P+S) \quad \text{subject to } 0 \leq P \leq (1-\rho)Y$$

$$(PW) \max_S \phi(Y-P-S) + \phi(P+S) \quad \text{subject to } 0 \leq S \leq \rho Y$$

where P and S are H and W 's savings decisions respectively. Thus household saving equals $(P+S)$. We first look at the case where the husband can impose his preferences. Formally this is equivalent to $\rho = 0$ (which, of course, implies that $S = 0$). Let P_2^H be the level of second period consumption chosen in this case. That is, P_2^H is the (interior) solution to:

$$(PD) \max_P v(Y-P) + \lambda v(P) \quad \text{subject to } 0 \leq P \leq Y.$$

Clearly, although P_2^H depends on the form of the utility function $v(\cdot)$ and the probability of survival λ we always have $P_2^H < Y/2$. The ratio of H 's preferred saving/income ratio turns up frequently enough to be given its own notation: $\pi = (P_2^H/Y)$. Conversely, if the wife can dictate (formally, $\rho = 1$) then consumption in each period is set to $Y/2$ so that her preferred saving rate is one half.

We first establish a result on the individual reaction functions (the proof and all subsequent proofs are given in the Appendix):

Lemma 1.

(a) The saving function of the wife is given by:

$$\hat{S}(P) = \max[\min\{\rho Y, 0.5Y-P\}, 0]$$

(b) The saving function of the husband is given by:

$$\hat{P}(S) = \max[\min\{(1-\rho)Y, \pi-S\}, 0]$$

Note that both these reaction functions are non-increasing in the other person's contribution to saving. Indeed, for interior solutions each person reduces his or her contribution to saving one-for-one with increases by the other person.

The analysis of any Nash equilibria is facilitated by the following result:

Lemma 2. At any Nash equilibrium $(\hat{P}, \hat{S})$, either H saves none of his income or W saves all her income.

Formally: $\hat{P}(\rho Y - \hat{S}) = 0$.

The intuition behind this result is trivial: since H discounts the future more than W he will never choose to contribute to future consumption unless W saves all her income. Notice that this does exclude the wife saving less than her total income ($\hat{S} < \rho Y$) nor the husband saving something ($\hat{P} > 0$).

We are now in a position to give household saving as a function of the intra-household income distribution.

Proposition 1. For given $(\rho, \lambda, v(.))$ there is a unique Nash equilibrium. Define π as given above. At the equilibrium $(\hat{P}, \hat{S})$:

(a) The wife saves all of her income ($\hat{S} = \rho Y$) if $\rho \leq 0.5$ and $Y/2$ otherwise.

(b) The husband saves none of his income ($\hat{P} = 0$) if $\rho \geq \pi$ and $(\pi - \rho)Y$ otherwise.

(c) Total saving at the Nash equilibrium $(\hat{P}, \hat{S})$ satisfies:

$$(\hat{P} + \hat{S}) = \pi Y \quad \text{for } \rho < \pi,$$

$$\begin{aligned}(\hat{P} + \hat{S}) &= \rho Y && \text{for } \pi \leq \rho \leq 0.5, \\(\hat{P} + \hat{S}) &= 0.5Y && \text{for } \rho > 0.5.\end{aligned}$$

Some remarks on this result are in order:

(a) One way to view this result is that the household behaves 'as though' it has a utility function $v(C_1) + \mu(\rho)v(C_2)$ where $\mu(\cdot)$ is non-decreasing and $\lambda \leq \mu(\rho) \leq 1$. In the conventional unitary model the discount function $\mu(\cdot)$ (and hence, the savings decision) is independent of the distribution of income in the household; this represents the source of the testable differences between the model presented here and the unitary model. Note that it is the husband's utility function that is used here to ensure that second period consumption equals P_2^H when μ equals λ^3 .

(b) In a single agent problem, marginal utilities are equalised over time. In the two person case treated here, if the wife has relatively little income ($\rho \leq \pi$) then the husband equalises his marginal utilities over time; conversely if the husband brings in less than half of the household income ($\rho \geq 0.5$) then the wife equalises her marginal utilities over time. If the wife has close to but less than half the household income ($\pi < \rho < 0.5$) then the result is a compromise; neither partner gets their own way entirely.

(c) The value of the husband's preferred saving rate ($= \pi$) depends on λ and his intertemporal substitution elasticity. If he has 'close to linear' preferences (that is, $v(\cdot)$ is 'not very concave') then π will be close to zero. Conversely, if he is very reluctant to substitute over time ($v(\cdot)$ is 'very concave' or preferences are close to Leontief) then π will be close to one half. Thus the width of the interval $[\pi, 0.5]$ depends on the husband's survival probability and his attitudes to intertemporal substitution.

(d) The reaction of household saving to a change in household income

³ If the wife also discounts the future then we need to take a weighted sum of the two utility functions.

only depends on who receives the extra income for the case $\pi < \rho < 0.5$ ⁴. Within this interval, any extra income going to the husband will be entirely spent whereas any extra income going to the wife will be saved. Thus for a given household income and a distribution of income in this range, the marginal propensity to consume (mpc) can be anything between zero and one depending on the distribution of any extra income. Note, as well, that this result persists even if there is near perfect sympathy between the two people so that λ is effectively close to one (see remark (i)). Thus even small differences in discount rates can lead to radically different mpc's over this interval. On the other hand, if λ is close to unity then the interval $[\pi, 0.5]$ will typically be small which somewhat offsets this sharp dichotomy.

For extreme ranges of ρ ($\rho \leq \pi$ and $\rho \geq 0.5$), however, the mpc is independent of who receives the income (and equal to π and 0.5, respectively). If the wife has relatively little of the household income and then receives an increase then she saves all of the increase. This leads the husband to decrease his saving in such a way that only π of the increase in income is saved. Conversely, if the wife has more than half of household income then she saves half of any extra she receives and increases her saving if the husband receives some extra income (all of which he spends).

(e) The responses given in proposition 1 are very general; they hold for all strictly increasing, strictly concave utility functions.

We now examine the case where there are some economies of scale but not so much that all consumption is effectively public. Following the suggestion in remark (iii) above we let an expenditure of C yield a utility of $\phi(C)$ for the wife in period 2 if the husband does not survive and $\phi(C/(1+\theta))$ if the husband does survive (where θ is between 0 and 1). The utility function for the husband is unchanged since he never consumes 'alone'. The utility function for the wife is⁵:

⁴ This is ignoring (second order effect) changes in π due to changes in Y .

⁵ In this example, it appears as though the wife is better off in utility terms if her husband dies. We could replace this rather heartless assumption by allowing for a positive additive term for the presence of the husband; this would not, of course, change any of the following analysis. On the other hand, the analysis might be considerably different if we allowed for state dependent

$$u_W = \phi(C_1) + \lambda\phi(C_2) + (1-\lambda)\phi((1+\theta)C_2).$$

Note that if we adopt a log utility function then we have:

$$u_W = \ln(C_1) + \ln(C_2) + (1-\lambda)\ln(1+\theta)$$

in which case the θ factor is irrelevant for W's decision and we have exactly the same set of outcomes as for the pure public good case already analysed.

If we are willing to assume that husband and wife have the same VN utility function then the previous result that the husband prefers a lower saving rate does go through even when there is some privateness.

Proposition 2. If husband and wife have the same within period utility functions ($v(C) = \phi(C)$) then the husband prefers a lower saving rate than the wife.

It follows that if there is some privateness and the within period utility functions are the same then we can derive a result similar to proposition 1 with π_W replacing 0.5 as the wife's preferred saving rate. This preferred saving rate can be greater than or less than one half but we always have $\pi_W > \pi$.

Remark (a) above leads on to a proposition concerning efficiency:

Proposition 3.

- (i) The Nash equilibrium level of household saving is Pareto efficient.
- (ii) Any efficient level of household saving can be rationalised as the outcome of our model for some value of ρ .

Part (i) of this proposition states that efficiency is an implication of our framework rather than an assumption, as in Chiappori (1988, 1992) and Browning and Chiappori (1994). Note that alternative values of (P,S) that give the same preferences.

level of household saving give the same utility outcomes as the Nash equilibrium so that the proposition applies only to total saving. Part (ii) implies that the rather unrealistic decision framework we have used can be justified if we assume that outcomes are efficient. For myself, I prefer the latter route: we have some reason to believe that household decisions are efficient (see, for example, Browning and Chiappori (1994) or Browning *et al* (1994) for some results on demand behaviour) so that this 'as if' story has some foundation. Note that once again part (ii) refers to household saving and not to the individual saving levels.

3. SOME ORDERS OF MAGNITUDE AND AN EXAMPLE

In this section I present some statistics and a simple example to give some idea of the magnitudes involved in the model developed above. The assumptions made will necessarily be somewhat crude; an empirical implementation of the model presented here would need to take more care of the many period problem and consider more than just expected values for lifetimes.

Table 1 presents the proportion of households in each of the three groups in proposition 1. These proportions are taken from Canadian Family Expenditure Survey (FAMEX) data for the years 1978, 1982, 1984, 1986 and 1990. The sample selection is for households headed by a married couple in which the husband was aged between 30 and 50 inclusive. The results are stratified by the presence of children in the household. The wife's relative income (ρ) is the ratio of her gross income from all sources to the sum of husband's and wife's gross income from all sources. A few households in which joint gross income was zero were excluded from the analysis but households in which one partner records zero income are not removed. The choice of $\pi = 0.4$ will be justified in the example below.

TABLE 1: PROPORTIONS OF HOUSEHOLDS IN DIFFERENT GROUPS

	couples without children	couples with children
wife's relative income high ($\rho > 0.5$)	22.2%	8.5%
wife's relative income medium ($0.4 \leq \rho \leq 0.5$)	22.4%	10.7%
wife's relative income low ($\rho < 0.4$)	55.4%	80.8%
total sample size	1716	7668

As we would expect, in the majority of households the wife's share of income is less than one half. Indeed, in the majority of cases her income is less than two thirds of her husband's. Nevertheless, there is a sizable proportion of households in which the wife has at least 40% of household income, particularly for those households without children.

As well as relative incomes, savings decisions will depend on the relative ages of the two partners. For the samples used in Table 1 the mean difference in age between husband and wife are 2.8 years and 2.7 years for the no children and children samples respectively. In Table 2 I present the distribution of the difference in ages.

TABLE 2: PROPORTIONS FOR AGE DIFFERENCES (%)

	wife older	same age	husband 1-3 years older	husband 4 or 5 years older	husband more than 5 years older
couples, no children	17.25	12.5	30.0	16.0	24.2
couples with children	11.8	14.6	39.8	16.0	17.8

As can be seen there are considerable numbers of households in which the husband is at least 4 years older than his wife. In the light of our model it might be expected that this would lead to disagreement about how much to save for retirement even if survival probabilities were the same for men and women.

To illustrate the differences in survival probabilities I take demographic information from the Canadian Life Tables for 1986. Given survival to age 65, the expected lifetime remaining is 14.4 years for a man and 18.6 years for a woman. For a woman aged 62 the expected remaining lifetime is 21 years⁶. Thus for a couple in which the wife is three years younger than the husband the difference in expected lifetime given that both survive until the husband is 65 is 6.6 years. To put it another way: a wife who is three years younger than her husband can expect to live 50% longer in the retirement period (here defined as the husband being over 65). For a couple in which the husband is currently aged 44 and the wife is aged 41⁷ the husband's discount factor is thus approximately two thirds. To show how this translates into different propensities to save we need to make some assumption about preferences.

We will follow conventional lines and assume that the husband has an iso-elastic utility function:

$$v(C) = \frac{C^{(1-\omega)}}{(1-\omega)} \quad (\text{with } v(C) = \ln(C) \text{ for } \omega = 0).$$

The parameter ω is the coefficient of relative risk aversion which is equal to minus the inverse of the intertemporal substitution elasticity (ise). Given this iso-elastic assumption we have that π is the solution to:

$$\pi^\omega - \lambda(Y-\pi)^\omega = 0.$$

For a value of λ (the husband's discount factor) of 2/3 and an ise of -1 (that is, $\omega = 1$), the husband sets $\pi = 0.4$. This means that the savings rate is 40% for households in which the husband receives income at least 50% higher than the wife and 50% for households in which the wife receives more income than her husband. Thus a wife who is three years younger than her husband will typically prefer saving for retirement at a rate about 25% above her husband's preferred savings rate. In figure 1 we present the household saving rate for three values of ω . As can be seen, the husband's preferred saving rate π is

⁶ This is not, of course, exactly equal to 21.6 (= 18.6 + 3) since there is some non-negligible probability of dying between age 62 and age 65.

⁷ These ages are chosen so that the wife's expected lifetime is approximately divided equally between the pre-retirement period (husband aged age 44 to 65 and wife aged 41 to 62) and the retirement period (husband aged over 65). This is assuming that the probability of death is zero for the wife before age 62.

lower the more he is willing to substitute across time⁸. Indeed, as the β approaches $-\infty$ the husband prefers to spend all current income. On the other hand, the lower the β , the smaller the interval in which the husband gets his own way.

In all of the above the only way to carry funds forward is by saving. We now examine the consequences of allowing for other financial instruments. The first we look at is the existence of a basic state pension. After that we examine the role of life insurance and annuities/occupational pensions.

⁸ Or the more willing he is to bear risk. As is well known, attitudes to risk and intertemporal substitution are inextricably linked in expected utility models with intertemporal additivity of the VN utility function.

4. THE EFFECTS OF A BASIC STATE PENSION

All industrialised countries have a basic pension scheme that pays out a pension that is independent of previous income to everyone over a certain age. This pension is set at a level that provides some socially accepted minimal living standard. Although the pension paid to one individual is, in reality, less than that paid to a couple, in our framework where all consumption is public, we shall not assume that this is the case. If all consumption is public then for a widow to have the same level of welfare as a couple she needs the same level of expenditure. Once again, we could allow for some economies of scale (see remark (iii) in section 2 and the discussion towards the end of section 2) but we prefer to stay with the simpler framework.

We assume that the government taxes each person an amount B in the first period and then pays out a pension $2B$ in the second period whether or not the husband survives. We denote $\frac{B}{Y}$, the benefit/household income ratio, by b . We assume that each person has income in period one at least as great as the pension contribution B . This avoids the problem of imputing the contribution if it is levied at a household level and one person has no income. Note that this pension scheme is actuarially fair.

We also assume that household income and the level of the pension are such that the basic pension ($2B$) is less than the husband would save for period 2. Formally we assume:

$$b \leq \rho \leq (1 - b) \quad \text{and} \quad 2b < \pi$$

Given that we are considering here a 'basic' pension these assumptions seem reasonable. It is important to recognise that in the unitary model such a pension would have no effects on saving - households would simply reduce their private saving by the amount of the pension. Since the effects of pensions on saving is the subject of continuing controversy this suggests that the unitary model does not command universal allegiance. In the non-unitary model considered here the provision of a state pension may affect the savings behaviour of the household.

The decision problems for husband and wife are now:

$$(PH') \max_P v(Y-2B-P-S) + \lambda v(P+S+2B) \quad \text{subject to } 0 \leq P \leq (1-\rho)Y-B$$

$$(PW') \max_S \phi(Y-2B-P-S) + \phi(P+S+2B) \quad \text{subject to } 0 \leq S \leq \rho Y-B$$

where the upper bounds on savings are given by gross income minus the pension contribution. As can be seen this is virtually identical to the original problem in section 2 if we replace (P,S) with 'effective' savings $(\tilde{P}, \tilde{S}) = (P+B, S+B)$:

$$(PH'') \max_{\tilde{P}} v(Y-\tilde{P}-\tilde{S}) + \lambda v(\tilde{P}+\tilde{S}) \quad \text{subject to } B \leq \tilde{P} \leq (1-\rho)Y$$

$$(PW'') \max_{\tilde{S}} \phi(Y-\tilde{P}-\tilde{S}) + \phi(\tilde{P}+\tilde{S}) \quad \text{subject to } B \leq \tilde{S} \leq \rho Y$$

The only difference from (PH) and (PW) in section 2 is that now the effective savings $\tilde{P}$ and $\tilde{S}$ are bounded below by B rather than zero. Thus the introduction of a basic state pension will only make a difference to second period consumption in the case where previously P was set equal to zero⁹. Specifically we have the following analogue to proposition 1:

Proposition 4. Second period consumption $\hat{C}_2$ at the Nash equilibrium satisfies:

$$\begin{aligned} \hat{C}_2 &= \pi Y & \text{for } \rho < (\pi-b), \\ \hat{C}_2 &= (\rho + b)Y & \text{for } (\pi-b) \leq \rho \leq (0.5-b), \\ \hat{C}_2 &= 0.5Y & \text{for } \rho > (0.5-b). \end{aligned}$$

Comparing this to proposition 1 we see that for households with low or high ρ ($\rho \leq \pi-b$ or $\rho \geq 0.5$) the path of consumption is unchanged. For other households, the level of second period consumption rises. The zone of non-dictatorship, previously $\rho \in (\pi, 0.5)$, shifts to the left by b to $(\pi-b, 0.5-b)$.

Thus the introduction of a state pension increases second period consumption

⁹ Since we have assumed that $2B$ is less than the husband's preferred second period consumption we have $2B < 0.5Y$ so that W never sets $\tilde{S}$ equal to B .

and benefits wives in families where she has relatively high income but less than her husband. In particular the introduction of a state pension makes some wives who had some power previously better off (those with $\rho \in (\pi, 0.5)$) as well as wives who previously had no effective say in the level of saving (those with $\rho \in (\pi - b, \pi]$).

One final corollary of proposition 4 is worth noting. The effect of the introduction of a state pension will be greater for households with lower household income, all other things (that is, λ and ρ) held constant. That is, the greater is b , the wider is the interval over which saving increases and the larger the increase in saving relative to household income.

5. ANNUITIES AND INSURANCE

In this section we drop the state pension but add further assets to the portfolios of husband and wife. The first new savings instrument that we consider is an actuarially fair annuity. This is an annuity that pays out λ^{-1} dollars for each dollar purchased in the first period, if the husband survives to the second period. Although the purchase of private annuities is relatively rare, the provision of work-related pensions is widespread. Employer provided pension plans for men that have poor (or no) survivor benefits act just like an annuity in this model. Thus the result given below is best seen as an analysis of the effect of employer-provided pensions¹⁰. As usual, there are all sorts of caveats to this. For example, there is usually an upper bound on the level of contributions to employee-provided pension schemes. It is also the case that there are pension schemes with good survivor benefits. Finally, pension schemes often give a higher return than private saving since they are 'tax-sheltered'. None of this is taken into account below.

The second new instrument is an actuarially fair insurance policy that

¹⁰ In the framework given here, wives are indifferent as between private savings and actuarially fair annuities for themselves. This obviously abstracts from any advantages or disadvantages that employee-provided pension may give. Since this analysis would be the same for singles as for couples these issues are not pursued here.

pays out $(1-\lambda)^{-1}$ dollars in the second period for each dollar purchased in the first period, if the husband dies. Since the annuity has a higher effective rate of return for the husband than private saving or insurance he will never choose to contribute to these. Conversely, the annuity has the same rate of return for the wife as private saving but a higher variance so that she never chooses to contribute to this. Thus we have:

Lemma 3. The wife never contributes to the annuity.
The husband never contributes to private saving or insurance.

This is convenient for the analysis since it means that we only need to examine the case where the husband has only one potential savings instrument - the annuity - and the wife has two other assets - insurance and private saving.

Given lemma 3 the decision problems for husband and wife are:

$$\max_P v(Y-P-S-I) + \lambda v\left(\frac{P}{\lambda} + S\right) \quad \text{subject to} \quad 0 \leq P \leq (1-\rho)Y$$

$$\max_{S,I} \phi(Y-P-S-I) + \lambda \phi\left(\frac{P}{\lambda} + S\right) + (1-\lambda)\phi\left(\frac{I}{(1-\lambda)} + S\right) \quad \text{subject to} \quad 0 \leq S + I \leq \rho Y$$

where P is now the husband's contribution to the annuity, S is the wife's contributions to private saving and I is her contribution to insurance. Given these decision processes we first show that the wife always desires purchases of insurance that just 'undo' the husband's contribution to annuities. This follows since, other things being equal, she prefers certain second period consumption. If she does not have enough income to do this fully then she spends all of her income on insurance:

Lemma 4. The wife sets $\hat{I} = \min(\rho Y, \frac{(1-\lambda)}{\lambda}P)$.

We also have an analog to lemma 2:

Lemma 5. At any Nash equilibrium $(\hat{P}, \hat{S}, \hat{I})$, either H saves none of his income or W spends all her income on saving or insurance. Formally: $\hat{P}(\rho Y - \hat{S} - \hat{I}) = 0$.

Thus we still have that the husband only saves if the wife is not contributing anything to current consumption. Given these two results we can show:

Proposition 5. There is a unique Nash equilibrium $(\hat{P}, \hat{S}, \hat{I})$. Define $\rho^* = 0.5(1-\lambda) < 0.5$.

For $0 \leq \rho \leq \rho^*$: $\hat{S} = 0$, $\hat{I} = \rho Y$ and $\hat{P} = \frac{\lambda(1-\rho)}{(1+\lambda)}Y$.

For $\rho^* < \rho \leq 0.5$: $\hat{P} = (0.5-\rho)Y$, $\hat{S} = \frac{(\rho-0.5(1-\lambda))}{\lambda}Y$ and
 $\hat{I} = \frac{(1-\lambda)}{\lambda}(0.5-\rho)Y$.

For $\rho > 0.5$: $\hat{S} = 0.5Y$, $\hat{I} = 0$ and $\hat{P} = 0$.

The details of proposition 5 are rather difficult to follow but the implications for consumption are very simple:

Corollary 1. At the Nash equilibrium we have:

(a) consumption in both periods and all states is $0.5Y$ if $\rho \geq \rho^*$.

(b) if $\rho < \rho^*$ then first period consumption and second period consumption if the husband survives equals $\frac{(1-\rho)}{(1+\lambda)}Y$. Second period consumption if the husband does not survive is equal to $\frac{\rho}{(1-\lambda)}Y$.

Thus for all but the lowest values of ρ (that is, for $\rho \geq \rho^*$) the household follows the wife's preferred consumption path. Fairly obviously, for any $\rho \geq \rho^*$ the wife prefers to have the annuity and insurance available. The converse is true for the husband: the availability of an annuity and insurance makes

him worse off as compared to the base case (strictly worse off for $\rho^* < \rho < 0.5$).

One notable feature of the result in proposition 5 is that the behavioural responses are independent of the specific functional forms of the two utility functions. Thus the presence of an annuity and an offsetting insurance policy removes the dependence of saving on the husband's attitudes to risk and intertemporal substitution. This should not, however, be taken too literally. By assuming a fixed real rate of interest and no other uncertainty than the date of death for the husband we have removed most of the variation that would give rise to precautionary motives and intertemporal substitution. Another feature of proposition 5 is that the value of ρ^* (the maximum expenditure on insurance) is quite small for plausible values of λ . For example, if the husband's survival probability is two-thirds then the household's insurance contributions are never more than 16.7% of household income.

In figure 2 we present a graphical illustration of the annuity/insurance case outcomes. We consider the case λ (the husband's survival probability) = 2/3, the value suggested by the analysis in section 3. As can be seen, insurance purchases are non-monotonic in the wife's share. The most striking feature of figure 2, however, is the extreme variability in the household portfolio as the wife's share of income changes. Households in which the wife has a low relative income buy some insurance and some annuity but save nothing whereas those households in which the wife earns more than half of income have only private savings.

From proposition 5 we have that very low relative income wives spend all of their income on insurance. However, as the wife's share of household income increases the husband decreases his contribution to the annuity and at some point the two terms in the $\min(\cdot)$ function in lemma 4 are equal; we denote this point ρ^* . At ρ^* the wife starts to contribute to private saving as well as to insurance. This causes the husband to decrease his annuity contributions at an even faster rate (and the wife to reduce her contributions to insurance

accordingly). Finally, when the incomes of the two partners are equal, the husband stops contributing anything to the annuity and the wife puts all of her income into private saving. Once her income exceeds that of her husband the wife holds her contribution to private saving constant at one half.

In the remarks following Corollary 1 we showed that the wife always prefers the annuity/insurance environment to the base case environment for $\rho \geq \rho^*$ and conversely for the husband. For low levels of ρ ($0 \leq \rho < \rho^*$), however, the welfare comparisons are not quite so simple. If the wife has no income then she has no second period consumption if the husband dies¹¹. Obviously she prefers the base case environment if $\rho = 0$ (given that utility approaches $-\infty$ as consumption approaches zero). However, the husband prefers the annuity/insurance environment if $\rho = 0$. This follows since he can always choose $P = \pi$ and receive a higher second period return if he survives. At $\rho = \rho^*$, however, the preferences for the base case *vis a vis* the annuity case are reversed. This raises the possibility that for some value of ρ between zero and ρ^* both partners may prefer not to have the annuity/insurance scheme. This in turn suggests that the Nash equilibrium may not be efficient for all values of ρ and all preferences. This turns out to be possible if the two partners have different VN utility functions but not otherwise:

Proposition 6.

For all preferences the Nash equilibrium levels of (P, S, I) are Pareto efficient for $\rho \geq \rho^*$.

If the two partners have the same VN utility function then the Nash equilibrium (P, S, I) is efficient for all values of ρ .

If the two partners have different VN utility functions then there may be values of the wife's share for which the household would prefer to ignore the existence of the annuity and insurance scheme. Until we have better estimates of the two utility functions and the parameters of the model it is difficult

¹¹ This dramatic outcome would be avoided if we allowed for a state pension of the sort discussed in the last period or if we allowed for some concern for the wife by the husband.

to ascertain how likely the existence of such an interval is.

Another issue that arises is whether the provision of annuities and insurance raises the level of saving as compared to the base case (that is the basic model of section 2). Since the level of saving is ambiguous in the annuity/insurance case it is easier here to concentrate on first period consumption. In figure 3 the first period consumption rates for the base case and the annuity case are graphed against the wife's share in household income. The parameter values chosen are $\lambda = 2/3$ and $\text{ise} = -0.5$ (which imply $\pi = 0.45$). It will be clear that the introduction of an annuity and insurance has an ambiguous effect on the first period consumption rate for very low values of the wife's share. Outside of this interval, however, the level of first period consumption is lower if we have actuarially fair annuities and insurance.

6. CONCLUSIONS

We have presented a model of household saving and portfolio behaviour in which we take explicit account of the possible differences between husbands' and wives' preferences for saving for the future. A number of conclusions emerge. First, household decisions depend on the relative incomes of the two partners; this is in marked contrast to conventional ('unitary') models of household behaviour. Second, we show that generally the outcome is efficient even though we adopt a non-cooperative framework. This suggests that our model is going to be consistent with a wide range of models that explicitly allow for intra-household differences in preferences. We also show that some conventional conclusions (for example, the neutrality of household saving decisions to the introduction of an actuarially fair state pension) need not hold if we move away from the unitary framework. Finally we present the implications for portfolio behaviour if we have annuities and insurance as well as private saving.

It is of considerable interest to trace out the implications of all of this for empirical work. Suppose that we wished to test whether household behaviour did conform to the model given here. One simple option is to

consider the effect of the wife's share in household income on current consumption. From the corollary to proposition 5, however, we see that current consumption is largely independent of the wife's share; at best we might only see some difference between the case where the wife has no income and the case in which she has a moderate to high share of household income. If we also allow for some caring and a state pension then even this may be masked. Moreover these differences will depend on the annuity and insurance options available to the household. Thus it seems unlikely that we will see simple effects of the wife's share on consumption. If we consider only households that do not have access to a company pension scheme then we might see more effect (see figure 3 again).

On the other hand, the model does provide a rich set of predictions for empirical models concerning portfolio choice and relative incomes. This is more difficult to look at than consumption but offers more chance of picking up any effects that are present. The most important prediction here is that portfolio choice will depend on the relative incomes of the two partners. We also have a non-monotonicity of insurance purchase with respect to the wife's share. Finally, there is a marked difference in the mix of private saving and annuity between households with a low wife's share and those households in which the wife receives a significant proportion of household income.

In the development of the model presented here many assumptions have been made. As indicated in the discussion in section two I believe that most of these assumptions could be relaxed at the cost of extra notation but with no change in the qualitative conclusions. On the other hand some of the assumptions may have more substantive impact. In particular, the assumptions of two periods, of exogenously given income and no uncertainty except for the husband's survival would not be directly applicable in empirical work. The 'driving' assumptions in the analysis - that husbands discount the future more than their wives and that there is less than perfect sympathy between husband and wife - would then be considered maintained assumptions that are to be tested by looking for the effects derived in this paper.

REFERENCES

- Browning, Martin; François Bourguignon, Pierre-André Chiappori and Valerie Lechene (1994), "Incomes and Outcomes: A Structural Model of Intra-Household Allocation", forthcoming, *Journal of Political Economy*.
- Browning, Martin and Pierre-André Chiappori (1994), "Efficient Intra-Household Allocations: A General Characterisation and Some Tests", mimeo, McMaster University.
- Chiappori, Pierre-André (1988), "Rational Household Labor Supply", *Econometrica*, 56, pp. 63-89.
- Chiappori, Pierre-André (1992), "Collective Labor Supply and Welfare", *Journal of Political Economy*.
- Thomas, Duncan (1990), "Intra-Household Resource Allocation: An Inferential Approach", *Journal of Human Resources*, Fall 1990, 25(4), pp 635-64
- Ulph, David (1988), "A General Non-Cooperative Nash Model of Household Consumption Behaviour", mimeo, University of Bristol.

APPENDIX

Proof of lemma 1.

I present here the proof of (a); the proof of (b) follows exactly similar lines. For the wife's decision problem we consider three cases:

(i) $\rho > 0.5$. This implies $\hat{S} < \pi Y$ and $P < 0.5Y$. If $\hat{S} = 0$ then:

$$\phi'(Y-P) \geq \phi'(P)$$

which implies $Y \leq 2P$ (since $\phi(\cdot)$ is concave). This contradicts $P < 0.5Y$. Thus $\hat{S} > 0$ so that we have an interior solution with:

$$\phi'(Y-P-\hat{S}) = \phi'(P+\hat{S})$$

which implies

$$\hat{S} = 0.5Y - P = \min(\rho Y, 0.5Y - P) = \max(\min(\rho Y, 0.5Y - P), 0)$$

since $0.5Y - P > 0$.

(ii) $\rho \leq 0.5$ and $0.5Y - P \geq \rho Y$. If $\hat{S} < \rho Y$ then:

$$\phi'(Y-P-\hat{S}) \geq \phi'(P+\hat{S})$$

which implies $\hat{S} \geq 0.5Y - P$ by concavity. Since $0.5Y - P \geq \rho Y$ this in turn implies $\hat{S} \geq \rho Y$, a contradiction. Hence

$$\hat{S} = \rho Y = \min(\rho Y, 0.5Y - P) = \max(\min(\rho Y, 0.5Y - P), 0)$$

since ρY and $0.5Y - P$ are both non-negative.

(ii) $\rho \leq 0.5$ and $0.5Y - P < \rho Y$. If $\hat{S} = \rho Y$ then:

$$\phi'(Y-P-\rho Y) \leq \phi'(P+\rho Y)$$

which implies $Y(1-2\rho) \geq 2P > Y(1-2\rho)$, a contradiction. Thus $\hat{S} < \rho Y$. If $\hat{S} > 0$ then once again we have $\hat{S} = 0.5Y - P$. Otherwise, $\hat{S} = 0$. Thus:

$$\hat{S} = \max(0.5Y - P, 0) = \max(\min(\rho Y, 0.5Y - P), 0)$$

since $\rho Y \geq 0.5Y - P$.

Proof of lemma 2.

If $\hat{P} > 0$ and $\hat{S} < \rho Y$ then:

$$v'(Y - \hat{P} - \hat{S}) \leq \lambda v'(\hat{P} + \hat{S}) \quad (\text{L2.1})$$

and

$$\phi'(Y - \hat{P} - \hat{S}) \geq \phi'(\hat{P} + \hat{S}) \quad (\text{L2.2})$$

From (L2.2) and the concavity of $\phi(\cdot)$ we have $(\hat{P} + \hat{S}) \geq (Y - \hat{P} - \hat{S})$. This implies $\lambda \geq 1$ in (L2.1) since $v(\cdot)$ is strictly concave and hence $v'(\cdot) > 0$. This contradicts $\lambda < 1$.

Proof of proposition 1.

Since the two agents have convex preferences and the strategy sets are convex we have that a Nash equilibrium exists. Uniqueness follows from the monotonicity in lemma 1.

From Lemma 2 we have $\hat{P} = 0$ or $\hat{S} = \rho Y$.

If $\hat{P} = 0$ then W's problem is:

$$\max_S \phi(Y-S) + \phi(S) \text{ subject to } 0 \leq S \leq \rho Y.$$

Since $\phi(\cdot)$ is unbounded below as C approaches 0 we can rule out $\hat{S} = 0$. We have:

$$\hat{S} < \rho Y \Rightarrow \phi'(Y - \hat{S}) = \phi'(\hat{S}) \Rightarrow \hat{S} = Y/2 \quad (\text{A1.1})$$

$$\hat{S} = \rho Y \Rightarrow \phi'(Y - \hat{S}) \leq \phi'(\hat{S}) \Rightarrow \hat{S} \leq Y/2 \Rightarrow \rho \leq 0.5 \quad (\text{A1.2}).$$

If $\hat{S} = \rho Y$ then H's problem is:

$$(\text{PA}) \quad \max_P v((1-\rho)Y - P) + \lambda v(\rho Y + P) \text{ subject to } 0 \leq P \leq (1-\rho)Y.$$

Setting $C = P + \rho Y$, this is equivalent to:

$$(\text{PA}') \quad \max_C v(Y - C) + \lambda v(C) \text{ subject to } \rho Y \leq C \leq Y$$

which is problem (PD) in the text, except that C is constrained $\geq \rho Y$. Now define:

$$V_H(C) = v(Y - C) + \lambda v(C).$$

This function is strictly concave and has a (unique) maximum at P_2^H . If $\pi = P_2^H/Y$ then $V_H(\cdot)$ has a maximum at πY which is feasible for (PA') if and only if

$\pi \geq \rho$. If $\pi < \pi$ then $V_H(C)$ is strictly decreasing for all feasible C . Thus we have:

$$\begin{aligned} \rho < \pi &\Rightarrow C = P_2^H \text{ is a solution to (PA')} \\ &\Rightarrow \hat{P} = (P_2^H - \rho Y) > 0 \text{ is a solution to (PA)} \end{aligned} \quad (A1.3)$$

$$\begin{aligned} \rho \geq \pi &\Rightarrow C = \rho Y \text{ is a solution to (PA')} \\ &\Rightarrow \hat{P} = 0 \text{ is a solution to (PA)} \end{aligned} \quad (A1.4).$$

CASE 1: $\rho > 0.5$.

We have $(\hat{S} < \rho Y)$ since (A1.2) rules out $(\hat{S} = \rho Y)$ and $(\hat{P} = 0)$ and (A1.4) rules out $(\hat{S} = \rho Y)$ and $(\hat{P} > 0)$. Thus $\hat{P} = 0$ and $\hat{S} = Y/2$, by (A1.1).

CASE 2: $\rho < \pi$.

If $(\hat{S} < \rho Y)$ and $(\hat{P} = 0)$ then $\hat{S} = Y/2$, by (A1.1) but this implies $\rho > 0.5$, a contradiction. Thus $\hat{S} = \rho Y$. From (A1.3) this gives $\hat{P} + \hat{S} = P_2^H$.

CASE 3: $\pi \leq \rho \leq 0.5$.

(A1.1) rules out $(\hat{S} < \rho Y)$ and $(\hat{P} = 0)$ since it implies $\rho Y > \hat{S} = (Y/2) \geq \rho Y$. Thus $S = \rho Y$. From (A1.4) we have $\hat{P} = 0$ so that $\hat{P} + \hat{S} = \rho Y$. ■

Proof of proposition 2.

The proof here is trivial. Since the wife's lifetime utility function is identical to that of her husband except that she gains additional utility from period 2 consumption (+ $(1-\lambda)\phi((1+\theta)(S+P))$ to be exact) she prefers a higher saving rate.

Proof of proposition 3.

(i) Let $(\hat{P}, \hat{S})$ be a Nash equilibrium. From proposition 1 we have $(\hat{P} + \hat{S}) \in [\pi Y, 0.5Y]$. If $(\hat{P} + \hat{S}) = \pi Y$ then any other (P, S) with $P + S \neq \pi Y$ makes the husband worse off. Conversely, if $(P + S) = 0.5Y$ then other (P, S) with $P + S \neq 0.5Y$ makes the wife worse off. For values strictly between πY and $0.5Y$, an increase in

$P+S$ makes the husband worse off and a decrease makes the wife worse off. Thus $(\hat{P}+\hat{S})$ weakly Pareto dominates any other (P,S) .

(ii) A level of household saving $(P+S)$ is efficient if and only if the household saving rate $(P+S)/Y \in [\pi, 0.5]$. By part (c) of proposition 1 we have that we can induce any such outcome by choosing ρ .

Proof of lemma 4.

Given any (P,S,I) with $S + I \leq \rho Y$, take the total differential of W 's utility function:

$$\begin{aligned} d\phi = & [-\phi'(Y - P - I - S) + \phi'(\frac{I}{(1-\lambda)} + S)]dI + \\ & [-\phi'(Y - P - I - S) + \lambda\phi'(\frac{P}{\lambda} + S) + (1-\lambda)\phi'(\frac{I}{(1-\lambda)} + S)]dS. \end{aligned}$$

If $I > \frac{(1-\lambda)}{\lambda}P$ then set $dI = -1$ and $dS = +1$. This gives:

$$d\phi = \lambda[\phi'(\frac{P}{\lambda} + S) - \phi'(\frac{I}{(1-\lambda)} + S)] > 0$$

since $\phi(\cdot)$ is strictly concave and $\frac{P}{\lambda} < \frac{I}{(1-\lambda)}$ by assumption. Thus W should decrease I and increase S until $I = \frac{(1-\lambda)}{\lambda}P$.

Conversely, if $I < \frac{(1-\lambda)}{\lambda}P$ then increasing I and reducing S by the same amount increases ϕ ; thus W should increase I and decrease S until $I = \min(\rho Y, \frac{(1-\lambda)}{\lambda}P)$.

Proof of lemma 5.

Suppose $P > 0$ and $S + I < \rho Y$. Then:

$$v'(Y - P - S - I) \leq \lambda v'(\frac{P}{\lambda} + S) \quad (L5.1)$$

and

$$\phi'(Y - P - S - I) \geq \lambda\phi'(\frac{P}{\lambda} + S) + (1-\lambda)\phi'(\frac{I}{(1-\lambda)} + S) \quad (L5.2)$$

Together these and strict concavity imply $I \geq \frac{(1-\lambda)P}{I}$ which in turn implies that $I = \rho Y$, from lemma 4. This contradicts $S + I < \rho Y$ and $S \geq 0$.

Proof of proposition 5.

From Lemma 5 we have $\hat{P} = 0$ or $\hat{S} + \hat{I} = \rho Y$.

If $\hat{P} = 0$ then W's problem is:

$$\max_S \phi(Y-I-S) + \lambda\phi(S) + (1-\lambda)\phi(S + \frac{I}{(1-\lambda)}) \quad \text{subject to } 0 \leq S + I \leq \rho Y.$$

Since $\phi(\cdot)$ is strictly concave we have:

$$\lambda\phi(S) + (1-\lambda)\phi(S + \frac{I}{(1-\lambda)}) < \phi(S + I) \quad \text{for } I > 0.$$

Thus setting $I = 0$ and increasing S by I gives a better outcome. From (A1.1) and (A1.2) in the proof of proposition 1 we have $S \leq 0.5Y$.

If $\hat{P} = 0$ then the first order condition for H's problem is $v'(Y-S) \geq v'(S)$ which implies $S \geq 0.5Y$.

Thus we have $\hat{P} = 0$, $\hat{I} = 0$ and $\hat{S} = 0.5Y$.

If $\hat{P} > 0$ then H's first order conditions (noting that $P = (1-\rho)Y$ is never chosen) is:

$$v'(Y - P - S - I) = v'(\frac{P}{\lambda} + S)$$

which implies:

$$P = \frac{\lambda}{(1+\lambda)}(Y - 2S - I) \tag{A5.1}$$

$$\text{CASE 1: } \rho Y \leq \frac{(1-\lambda)}{\lambda}P.$$

From lemma 4 we have $I = \rho Y$ so that $S = 0$ from lemma 5 and $P > 0$. Thus:

$$P = \frac{\lambda}{(1+\lambda)}(1-\rho)Y.$$

Given this some elementary manipulation gives $\rho \leq 0.5(1-\lambda)$, the condition given in the proposition.

CASE 2: $\rho Y > \frac{(1-\lambda)}{\lambda}P$.

From lemma 4 and 5 we have $\hat{I} = \frac{(1-\lambda)}{\lambda}P$ and $\hat{S} = \rho Y - \frac{(1-\lambda)}{\lambda}P$. Substituting for S and Y in (A5.1) and solving, we have:

$$P = (0.5-\rho)Y.$$

Given this we can show that $\rho > 0.5(1-\lambda)$, the condition required in the proposition. Substituting P into the expression for $\hat{I}$ and $\hat{S}$ gives the forms of the proposition.

Proof of proposition 6.

If $\rho \geq \rho^*$ then consumption in all states and all time periods is $0.5Y$. Clearly this cannot be changed without making W worse off whatever preferences W and H have.

If both partners have the same the same VN utility function $v(C)$ then the utility differences between the base case and the annuity case are given by:

$$\Delta_H = v(1-\pi) + \lambda v(\pi) - (1+\lambda)v\left(\frac{1-\rho}{1+\lambda}\right)$$

and

$$\Delta_W = v(1-\pi) + v(\pi) - (1+\lambda)v\left(\frac{1-\rho}{1+\lambda}\right) - (1-\lambda)v\left(\frac{\rho}{1-\lambda}\right).$$

Since Δ_H and Δ_W are both continuous and strictly monotone in ρ , with Δ_W (Δ_H) strictly decreasing (respectively, increasing) we have that $\Delta_W = \Delta_H$ for exactly one ρ . If both H and W prefer the base case then $\Delta_W = \Delta_H > 0$ for that ρ . The equality gives:

$$(1-\lambda)v(\pi) = (1-\lambda)v\left(\frac{\rho}{1-\lambda}\right) \Rightarrow \pi = \frac{\rho}{1-\lambda}.$$

The inequality $\Delta_H > 0$ gives:

$$\frac{1}{(1+\lambda)}v\left(\frac{1-\lambda-\rho}{1-\lambda}\right) + \frac{\lambda}{(1+\lambda)}v\left(\frac{\rho}{1-\lambda}\right) > v\left(\frac{1-\rho}{1+\lambda}\right).$$

From the concavity of $v(\cdot)$ this gives:

$$v\left(\frac{1-\lambda-\rho+\lambda\rho}{(1-\lambda)(1+\lambda)}\right) > v\left(\frac{1-\rho}{1+\lambda}\right) \Rightarrow 1-\lambda-\rho+\lambda\rho > (1-\rho)(1-\lambda),$$

a contradiction. Thus we cannot have $\Delta_W = \Delta_H > 0$ which means that whenever one person prefers the base case the other person does not.

Figure 1: Saving rates for base case

Figure 2: Saving rates with annuities and insurance

Figure 3: First period consumption rates

Recent McMaster University Economics Working Papers

(To obtain copies, write to: Secretary, Working Papers, Department of Economics, McMaster University, Hamilton, Ontario, Canada L8S 4M4. A charge of \$3 per paper will be levied on orders from institutions that do not have an arrangement for the exchange of working papers. Orders from individuals will be met free of charge, supplies permitting.)

No. 93-07	On Canadian Wage Inequality: the 1970s and 1980s
J.B. Burbidge, L. Magee, and A.L. Robb
No. 93-08	Dual Approaches to Utility
Martin Browning
No. 93-09	Re-Examining the Assumption that Children are Exogenous to
Female Labor Supply
Xiaodi Xie
No. 93-10	Incomes and Outcomes: A Structural Model of Intra-Household
Allocation
Martin Browning, Francois Bourguignon, Pierre-Andre Chiappori
and Valerie Lechene
No. 93-11	On John Rae's Economics
Syed Ahmad
No. 93-12	The Measurement of Unemployment: An Empirical Approach
Stephen R.G. Jones and W. Craig Riddell
No. 94-01	Is Strike Behaviour Cyclical?
Alan Harrison and Mark Stewart
No. 94-02	Efficient Nash Equilibria in a Federal Economy with Migration
Cost
Gordon M. Myers and Yorgos Y. Papageorgiou
No. 94-03	Boundary Effects and Voluntary Contributions to Public Goods
Kenneth Chan, Rob Godby, Stuart Mestelman and Andrew Muller
No. 94-04	On the Use of Sampling Weights when Estimating Models with
Survey Data
L. Magee, A.L. Robb and J.B. Burbidge
No. 94-05	Trade Unions as Discriminating Monopolists: Theory and
Evidence
Peter Kuhn and Arthur Sweetman
No. 94-06	The Saving Behaviour of a Two Person Household
Martin Browning
No. 94-07	Efficient Intra-Household Allocations: A General
Characterisation and Empirical Tests
Martin Browning and Pierre-Andre Chiappori

QSEP Research Reports

Number	Title	Author(s)
295	Competitive Bidding for Independent Power: Developments in the US	M. Kliman
296	Modelling the Structure of For-Hire Motor Carrier Revenues, by Commodity, and Community with an Example from Ontario Under Regulation, 1987	C.G. Woudsma P.S. Kanarogiou
297	Structural Change and Economic Development of Egypt Between Planning and the Open Door Policy	M.A. Elkhafif A.A. Kubursi
298	Aboriginal Land Use and Harvesting in the Moose River Basin: A Historical and Contemporary Analysis	P.J. George F. Berkes R.J. Preston
299	Management Education in the People's Republic of China	M.W.L. Chan
300	A Binary Choice Model with Coefficients that are Elasticities	L. Magee
301	Teachers and the Birth Rate: The Demographic Dynamics of a Service Population	F.T. Denton C.H. Feaver B.G. Spencer
302	The SHARP System for Health Care Planning: A Description and an Application Relating to the Reduction of Ontario Medical School Enrolment	F.T. Denton A. Gafni B.G. Spencer
303	The SHARP Way to Plan Health Care Services: A Description of the System and Some Illustrative Applications in Nursing Human Resource Planning	F.T. Denton A. Gafni B.G. Spencer
304	Data Collection and Macroeconomic Control: Some Theory of the Optimum Sample Size in a Repeated Survey	F.T. Denton
305	A Framework for Regional Modeling and Impact Analysis -- An Analysis of the Demand for Electricity by Large Municipalities in Ontario, Canada	C. Hsiao D.C. Mountain
306	Wildlife Harvesting and sustainable Economy in the Mushkegowuk Region of the Hudson/James Bay Lowlands	F. Berkes P.J. George R.J. Preston A. Hughes J. Turner