**THE INTEREST ELASTICITY OF SAVING
AND THE FUNCTIONAL FORM OF
THE UTILITY FUNCTION**

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ABSTRACT

This paper deals with the important issue of the elasticity of saving with respect to the interest rate. Michael Boskin, and more recently, Lawrence Summers, have argued that saving is much more interest-elastic than economists have generally believed, and as a consequence that the dynamic efficiency losses from capital income taxation are much higher than they were previously thought to be. In Summers' life-cycle simulation model an increase in the interest rate depresses the present value of future labour income and this leads to declines in the consumption of younger cohorts, more saving, and a higher capital stock. These results are established with a CES utility function. I argue that it is reasonable to introduce minimum consumption levels into the model, as in the stone-Geary utility function, and that when this is done, younger cohorts do not decrease their consumption by as much in response to an interest rate increase. I show that the interest elasticity of saving is significantly reduced and may even be negative.

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I. INTRODUCTION

The interest elasticity of the aggregate saving rate is a key parameter for a number of important questions concerning the government's influence on capital formation. Recently, significant attempts have been made to examine and estimate the effects of interest rate changes on aggregate saving. Hamburger (1967) and Wright (1969) attempted to estimate the interest elasticity of saving. Subsequent studies have reported divergent results, and with the publication of Boskin's (1978) controversial study, the profession has moved away from a consensus on this issue¹.

In a challenge to the widely held view that the elasticity is low, Summers (1981) claimed to have formulated a realistic life-cycle model yielding a large and positive interest elasticity of saving. In fact, he made the strong claim that "the theory when formulated realistically implies interest elasticities well in excess of unity" (1981, p. 534). The claim has been questioned by Evans (1983), who showed that with negative time preferences the interest elasticities are far lower. Furthermore, he incorporated inter-generational transfers into the model and demonstrated

that negative elasticities are not implausible.

Seidman (1983) introduced bequests into the model and obtained results that are qualitatively similar to those of Summers.

The studies by Summers (1981), Evans (1983) and Seidman (1983) share a common feature in that each employed a standard homothetic CES utility function (hereafter referred to as SHCES) to represent the preferences of the individual. None of these authors addressed the issue of the interest elasticity of saving in relation to alternative functional forms of the utility function.

This paper raises further questions about Summers' claim by showing that it does not hold up under reasonable alternative specifications of the utility function holding parameters constant. To make my point, a quasi-homothetic constant elasticity of substitution utility function (hereafter referred to as QHCES), which is more general than the function employed in previous studies, is proposed. It is found that savings are indeed sensitive to the functional form, and that a negative interest elasticity is not implausible on a priori grounds.

A multiperiod life-cycle model of consumption

is developed in Section II. In Section III, some simulation experiments based on the model are reported and the results discussed. A brief summary and statement of conclusions is provided in Section. IV.

II. AN OUTLINE OF THE MODEL

In this section of the paper, I outline a multi-period life-cycle model that contains the essentials of Summers' model but extends it to allow for subsistence consumption. A utility function for an individual consumer which is particularly well suited to the present purpose is obtained from a quasi-homothetic function of the form²:

$$(1) \quad U_{\tau} = \begin{cases} \sum_{t=\tau}^T \frac{(C_t^*)^{\delta}}{\delta} (1 + \rho)^{\tau-t} & (\text{if } \delta < 1 \text{ and } \neq 0) \\ \sum_{t=\tau}^T \log C_t^* (1 + \rho)^{\tau-t} & (\text{if } \delta = 0) \end{cases}$$

where $C_t^* = (C_t - \gamma_t) > 0$,

$$\gamma_t \geq 0, \quad \sigma = \frac{1}{1-\delta} ;$$

and T = age to which the individual will live,
 which he knows with certainty;
 C_t = total consumption at age t ;
 γ_t = subsistence consumption at age t ;
 σ = intertemporal elasticity of substitution between consumption in any two periods;
 ρ = rate of time preference;
 τ = individual's present age.

The utility function given by equation (1) is the generalized CES function. This function has some desirable properties in that it does not restrict preferences to be homothetic from the origin and under proper specification of the subsistence consumption parameter it yields the SHCES function. In spite of this, the function appears to have attracted little attention from researchers dealing with the allocation of consumption and saving over time, as an alternative to the SHCES function. The utility function employed in Summers' paper is of the latter kind. In the present paper, the allowance for subsistence consumption levels in the life-cycle model is intended to capture

the notion that at some ages individuals may have consumption that can not be redistributed or sacrificed over time for purposes of life-cycle optimization.

Following Summers, I assume each individual to be engaged in life-cycle consumption planning, with no inheritance or bequest. I assume also that the individual's wage rate grows at an exogenous rate g per period until a fixed age of retirement. The individual maximizes (1) subject to the following budget constraint:

$$(2) \quad \sum_{t=\tau}^T C_t (1+r)^{\tau-t} = a_{\tau} + \sum_{t=\tau}^R W_t (1+r)^{\tau-t}$$

where r is the after-tax rate of return on savings (proportional taxation being assumed), W_t is the wage rate net of tax, a_{τ} is net worth at the beginning of a year, and R is a fixed age of retirement from the work force.

The first-order condition yields the following:

$$(3) \quad C_{t+1}^* = [(1+r)/(1+\rho)]^{\sigma} C_t^*$$

Equation (3) determines the shape of the discretionary consumption profile, but not its level. It is obvious that the partial elasticity of substitution between C^* and C_{t+1}^* becomes smaller the stronger the desire of the individual to smooth his discretionary consumption profile. For a given value of σ , however, the rate of discretionary consumption increases with an increase in the effective interest rate and decreases with an increase in the rate of time preference. To obtain an expression for discretionary consumption C_τ^* equations (2) and (3) are solved simultaneously. A simple manipulation of the result yields a consumption level for $t = \tau$:

$$(4) \quad C_\tau^* = \left\{ \sum_{t=\tau}^4 [(1+r)/(1+\rho)]^{(t-\tau)\sigma} (1+r)^{\tau-t} \right\}^{-1} (Y_\tau - V_\tau)$$

$$\text{where } Y_\tau = a_\tau + \sum_{t=\tau}^R W_t (1+r)^{\tau-t}$$

$$\text{and } V_\tau = \sum_{t=\tau}^T Y_t (1+r)^{\tau-t}$$

For the level as well as the optimal path of consumption given by (3) to be fully specified, we need to specify a mechanism by which subsistence consumption levels are determined over the life-span of the individual. A simple assumption for illustrative purposes is that they are constant (i.e., $\gamma_1 = \gamma_2 = \dots = \gamma_T$)³.

There are T cohorts alive at any point in time. I assume population grows at an annual proportionate rate n . Aggregate consumption is computed by summing over cohorts, and the corresponding aggregate saving and saving rate out of total income can then be calculated.

III. PARAMETERIZATION OF THE MODEL

I use the theoretical model presented above as the basis for a series of simulations. To provide a simple and workable framework for the analysis, I assume that individuals plan their consumption path at age 21, that they re-optimize each year thereafter, that they retire from the labour force at age 60, and that they die at age 70. The model requires additional assumptions with regard to the lifetime wage profile,

population growth rate, and interest and time preference rates, as well as the assignment of values to γ and δ . As indicated the subsistence level of consumption is assumed constant over time.

To generate values for the γ 's, I followed the procedure described below⁴:

My aim is to provide a counter example to Summers' claim that the interest elasticity is large and positive. To do so, I assume initially that the individual maximizes the SHCES function subject to the life-time budget constraint (2). The parameter δ is set in the range of 0.5 to -2.0. Furthermore, each individual in the labour force is assumed to receive a wage rate that grows at a rate of 2 per cent per annum because of productivity growth, population is assumed to grow at 1.5 per cent per annum. The effective rate of return on savings and the rate of time preference are set equal to 6 per cent, yielding an optimal path of consumption which is perfectly horizontal. Let the constant consumption level be denoted by $\bar{C}$ and then assume subsistence consumption to be a fraction of $\bar{C}$:

$$(5) \quad \gamma_t = \psi \bar{C}_t \quad \forall_t$$

where $\psi \geq 0$.

In the simulation reported below, I experiment with ψ values of 0.00, 0.33, 0.66 and 0.90.

The values of the γ 's obtained in the above manner are imposed as the lower bounds on consumption in maximizing the QHCES utility function given by equation (1) subject to the budget constraint (2). Following Summers, the rate of time preference is set at 3 per cent per annum, while the effective rate of return on savings is allowed to take values ranging from 4 to 8 per cent per annum.

The resulting interest elasticities of saving and the saving rates (defined as the ratios of aggregate saving to total income) are reported in Table 1. As pointed out by Evans, the change in the interest rate that led to a change in saving can be viewed as arising from a cut in the tax rate on capital income. The interest elasticities of savings are computed at various values of the interest rate.

The interest elasticities and saving rates corresponding to those of Summers can be read from the first row of each block in Table 1 (the rows with $\psi = 0$) for various values of r . Within each block, it is clear that the elasticity declines as the subsistence level increases. The results in Table 1 indicate that Summers' conclusion is by no means insensitive to changes in functional specification of the utility function and that negative interest elasticities of saving are not implausible.

The incorporation of subsistence consumption into Summers' model implies that the individual is constrained in his ability to trade off current and future consumption in response to interest rate changes. The intertemporal substitution effect is limited to discretionary consumption in the case of the QHCES utility function, and thus may be weaker than the corresponding effect with the SHCES function. The income effect, on the other hand, depends on whether the individual dissaves or not when young, and the size of his net worth. This, in turn, depends on the level of subsistence consumption. With significant dissaving in youth, the income effect of an interest rate change, which works to

TABLE 1

INTEREST ELASTICITIES OF SAVING AND SAVING RATES

Block	Values of δ ψ		VALUE OF r					
			0.04		0.06		0.08	
			Saving Elasticity	Saving Rate	Saving Elasticity	Saving Rate	Saving Elasticity	Saving Rate
#1	0.5	0.00	3.97	0.11	2.50	0.24	2.60	0.29
		0.33	3.86	0.08	2.48	0.19	2.57	0.25
		0.66	3.55	0.04	2.42	0.11	2.50	0.17
		0.90	2.05	0.02	2.01	0.03	1.91	0.05
#2	0.0	0.00	3.55	0.06	2.14	0.13	2.06	0.18
		0.33	3.33	0.03	2.11	0.09	2.05	0.14
		0.66	2.67	0.02	2.01	0.05	2.00	0.07
		0.90	-0.66	0.01	-0.53	0.01	1.19	0.01
#3	-0.5	0.00	3.25	0.04	1.94	0.09	1.76	0.13
		0.33	2.90	0.03	1.90	0.06	1.75	0.09
		0.66	1.86	0.01	1.72	0.03	1.73	0.04
		0.90	-3.65	0.004	*	*	*	*
#4	-1.0	0.00	2.96	0.03	1.82	0.06	1.60	0.09
		0.33	2.49	0.02	1.78	0.04	1.60	0.06
		0.66	1.06	0.01	1.45	0.02	1.58	0.02
		0.90	-7.05	0.002	*	*	*	*
#5	-2.0	0.00	2.43	0.02	1.65	0.04	1.44	0.06
		0.33	1.70	0.01	1.51	0.02	1.44	0.03
		0.66	-0.60	0.01	0.47	0.01	1.43	0.01
		0.90	-15.67	0.001	*	*	*	*

NOTES: The computations assume $T = 50$, $R = 40$, $g = .02$, $n = .015$, and $\rho = .03$. The saving rate is calculated relative to total income⁵.

What Summers calls γ in his paper is identical to δ in the present study.

Subsistence consumption level is defined as a fixed proportion ψ , of total consumption $\bar{C}$, where $\psi \geq 0$.

Asterisks indicate negative saving rates.

lower consumption may not be obvious in the presence of subsistence consumption. Thus, the net effect of a change in interest rate on consumption is much less certain in the QHCES than in the SHCES case.

IV. CONCLUSION

The purpose of this paper has been to investigate the intertemporal relationship between saving and the rate of interest. The claim that a realistically formulated life-cycle model necessarily yields large positive interest elasticities of saving does not survive generalization of the model.

The homotheticity assumption made by Summers (1981) was replaced in this paper with the more general assumption of quasi-homotheticity and it was demonstrated that both large positive and large negative elasticities can result, depending on the underlying assumptions. (The plausibility of a negative response of saving to a change in the interest rate was shown previously by Evans (1983).) Holding parameters constant, I have shown that Summers' claim of high interest elasticities well in excess of unity is not robust in the face of changes in the functional form of the utility function.

FOOTNOTES

1. Gylfason (1981) provides a comprehensive summary of the size and magnitude of the interest elasticity of saving (consumption) parameters estimated in earlier studies. Carlino (1981) discusses possible causes of the differences.
2. Quasi-homotheticity is more general than homotheticity. Although it has been widely used in consumer theory at the static level, only a few researchers have exploited it at the intertemporal level. Among them are Heien (1972), Betancourt (1973), Somermeyer and Bannink (1973), Ashenfelter and Ham (1979), and Biorn (1980).
3. The assumption that subsistence consumption does not change over the life-span of the individual does not imply that individuals of different generations have identical subsistence consumption levels.
4. Note that the interest elasticity of saving may be sensitive to the specification of the γ 's.
5. Summers (1981) calculated saving rates relative to labour income rather than total income.

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